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George J Klir

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# FLAWS OF FORM

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G. Spencer Brown's book *Laws of Form* has been enjoying a vogue among social and biological scientists. Proponents claim that the book introduces a new logic ideally suited to their fields of study, and that the new logic solves the problems of self-reference. These claims are false. We show that Brown's system is Boolean algebra in an obscure notation, and that his "solutions" to the problems of self-reference are based on a misunderstanding of Russell's paradox.

INDEX TERMS: Spencer Brown's calculus, Boolean algebra, self-reference, Russell's paradox.

## NTRODUCTION

n this paper we investigate the logic described by G. Spencer Brown in his book *Laws of Form*.[2] All references to Brown are to this book. Brown claims to have invented a new logic and a superficial perusal of his book, with its unusual notation, certainly suggests that something different from our usual logic is being described. It is our contention, however, that Brown has merely reinvented Boolean algebra but in an obscure notation. In the first section of this paper we discuss Brown's notation and show how it allows him to obtain implicit axioms "for free." We also show that using Brown's explicit and implicit arithmetic axioms we obtain exactly Boolean arithmetic. In the second section we consider Brown's algebraic axioms and show that they are synonymous with the axioms for Boolean algebra.

A fascinating aspect of Brown's work is his treatment of inconsistent equations. Brown's proponents claim that this treatment solves Russell's paradox and successfully handles problems of self-reference (e.g., Varela in his 1975 work[26] in which he extends Brown's system by adding a third value, and Howe and von Foerster in their 1975 comments on Varela's work[8]). We investigate these claims in the third section of this paper. We argue that what is correct about Brown's treatment is a reinvention of part of the theory of sequential machines. Further we show that Brown cannot solve Russell's paradox since his logic is too weak to even state the paradox. We conclude with some general discussion of the meaning of paradox.

## 1 NOTATION AND THE ARITHMETIC AXIOMS

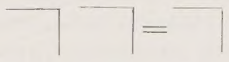
An arithmetic is an equational system (of rules, axioms plus theorems, etc.) containing no variables.

In this section we will show that Brown's arithmetic looks very different from Boolean arithmetic but, in the end, comes to the same thing.

### *1.1 The Form of Brown's Original Axioms*

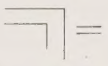
Brown admits to only two axioms:

(F 1) The Law of Calling



(The value of a call made again is the value of the call.)

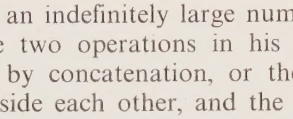
(F 2) The Law of Crossing

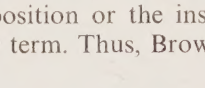


(The value of a crossing made again is not the value of the crossing.)

The equals signs in Brown's system are intended as signs of ordinary mathematical identity. He appears to employ only one other symbol, the right corner, "¬", but in fact employs at least one other, a blank. Moreover, as we shall see, both the corner and the blank must be available in an indefinitely large number of sizes.

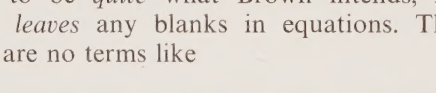
There are two operations in his system. one represented by concatenation, or the writing of terms alongside each other, and the other represented by superimposition or the inscribing of a term above another term. Thus, Brown has terms like





and

The oddest thing about these axioms is the absence of any apparent symbol on the right hand side of (F 2). Here, one might presume, a blank character is intended. In fact, this does not seem to be *quite* what Brown intends, for he never *leaves* any blanks in equations. That is, there are no terms like



What is going on here? First of all, the laws of his arithmetic make all terms with a blank equal to similar terms without a blank, so Brown can omit blanks without disturbing the truth-values of his equations. But beyond this, Brown *thinks* that he is using only one symbol; he thinks that his language has only one character, the corner, or that if there is a blank in his system, there is only one token of it, simultaneously occupying all the uninscribed paper on which equations are written. While blank characters can be used in a legitimate way, this is not it. In thinking of his language as containing only one contentful character, Brown violates the most basic truth of information theory: at least two symbols are required to convey any information.

We are forced, then, to an interpretation o (F 2) as containing a *blank character* on the right Under this interpretation, what looks like a pe culiar incomplete equation is merely inper spicuous, since non-digital readers, like people find it difficult to recognize blanks. Of course device that reads in discrete pieces, like a digita computer, functions well using a blank characte as an unmarked state, even though humans d not. For instance, humans may ask how man blanks they are to notice when they read (F 2 One? One hundred? We must ask how *big* th blanks are. Noting that the corners vary in size we must presume that the blanks can too. For digital reader, answers to all those question would be presupposed.

The problems caused by using a blank charac ter are compounded by Brown's use of position instead of symbols to indicate operations. Thu any space of paper left unfilled might indicat any manner of operations applied to sequences o whatever we take the blank character to repre sent. In fact, Brown treats blank spaces as in finitely ambiguous in just this way, and—perha brilliantly but certainly illegitimately—thus o tains additional axioms "for free".

Before considering his axioms, we will e change Brown's notation for a more conventiona one. Now proponents of *Laws of For* sometimes appear to think that a deep and satisfy ing truth is revealed through the shapes Brown's symbols, as adherents of religions ofte do about the symbols they use, but the me aphysical significance its advocates seem to deriv from Brown's notation should not be confuse with its mathematical substance. From a ma thematical point of view, notation is insubstan tial: notation can be clear or confusing, redund ant or concise, convenient or awkward, but cannot change the subject the language is used t inform us about. Brown merely replaces th ordinary ideographic notation of mathematic with a positional or analytic notation. (Se Lyons[16]). Again (contra Kauffman[10]), the shap nature, etc., of the signs one chooses to conve information with are irrelevant to the mathemat cal content of what is conveyed. What *is* relevan is the meaning of the symbols and the way the are combined. Brown's system is the same as an other system of axioms employing logical sym bols with the same meaning (here, only identity and the same configuration of axioms, albe expressed with different characters.

Note that we are not concerned here about the *interpretations* of the axioms, the things for which the non-logical constants are intended to stand, but only about the abstract axiom system itself. For example, Boolean algebra can be interpreted as an algebra of switching circuits, sets, or truth functions. But the language, axioms, and theorems used to describe these interpretations all remain the same. Thus, even if Brown *has* discovered a new interpretation for Boolean algebra, he has not discovered a new mathematical system (contra Whyte[29]). Now, since we will show that Brown's systems *are* Boolean, he has at best discovered a new interpretation for such systems. But Brown's informal remarks, such as those associated with the axioms above, cannot be regarded as providing *any* interpretation for his system; they are simply too incredibly vague and abstract. For example, about the symbol "$\neg$", he says:

> KNOWLEDGE
> Let a state distinguished by the distinction be marked with a mark
>
>
>
> of distinction. (page 4)

### 1.2 *The Axioms in Conventional Notation*

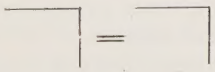
To return to the main thread of our argument, we have argued that we may obtain a translation of Brown's axioms into a more conventional language. By allowing "1" to replace "$\neg$", "0" to replace the blank, "$\vee$" for concatenation and "$\oplus$" for superimposition, the axioms appear as

(F 1*) $\quad 1 \vee 1 = 1$

(F 2*) $\quad 1 \oplus 1 = 0$

and when "$\vee$" and "$\oplus$" are taken as inclusive and exclusive Boolean addition, we have two familiar laws of Boolean arithmetic.

The truly creative, but, we maintain, suspicious aspect of Brown's axioms is in his use of the identities

(F 3) 

and

(F 4) 

As we have pointed out, blanks and operations performed on blanks are undetectable, so (F 3) and (F 4) represent, among a multitude of equations, the fundamental ones

(F 3*) $\quad 1 \vee 0 = 0 \vee 1 = 1 \oplus 0 = 0 \oplus 1 = 1$

and

(F 4*) $\quad 0 \vee 0 = 0 \oplus 0 = 0$

The conceptual or methodological confusion involved in deriving (F 3*) and (F 4*) from (F 3) and (F 4) is that (F 3) and (F 4) are tautological identities (assuming we can make sense out of (F 4) at all). That is, (F 3) and (F 4) *must* be true, simply as a matter of logic, while (F 3*) and (F 4*) are no such thing, but instead give us information about the working of the functions "$\vee$" and "$\oplus$". Of course it is impossible to derive non-tautological information from a tautology, so the fact that Brown seems to have done so is highly curious.

In any case, equations (F 1*) through (F 4*) are translations of equations true in Brown's system, and as these equations serve to uniquely determine the operation of "$\vee$" and "$\oplus$" on "1" and "0", they complete the Boolean arithmetic for inclusive and exclusive disjunctions. Now it is well known (for example, Klir, 1969) that these two functions, together with the constants, allow us to define all other Boolean functions. Since Brown's laws themselves are familiar laws of Boolean arithmetic, it is clear that the two systems, except for notation, are the *same*. This interpretation of Brown's arithmetic has been given by Banaschewski.[1]

Finally, let us note that another translation, perhaps truer to what we suspect Brown's intentions to be, is possible. This translation involves treating the blank as the only constant, concatenation as the only binary function indictator, and treating the corner as a unary function symbol instead of as a constant, writing it over the constants to which it is being applied. Thus any occurrence of the corner by itself in an equation would entail the presence of a blank within it for the corner to operate on.

Assigning these grammatical categories to his symbols, "$\neg$" can now be translated as the Boolean complement operation. But because complement is definable in terms of exclusive or and one as

$$-a = a \oplus 1,$$

the two translations are equivalent.

This second translation may be closer to what Brown has in mind for "⌐", because he sometimes seems to want to use this symbol to stand for an *instruction*—which is to say, a function—rather than an *object*. Contrast, for instance: "Let any token be intended as an instruction to cross the boundary of the first distinction", on page 5, with the name-like suggestions about the meaning of "⌐" on page 4, quoted above.

In this guise, Brown's axioms appear as follows:

$$-0 \vee -0 = -0$$

$$--0 = 0$$

$$-0 \vee 0 = 0 \vee -0 = -0$$

$$0 \vee 0 = 0,$$

and when the definition

$$1 = -0$$

is employed, they can be rewritten as

(F 1*) $1 \vee 1 = 1$

(F 2*) $-1 = 0$

(F 3*) $1 \vee 0 = 0 \vee 1 = 1$

(F 4*) $0 \vee 0 = 0$

These last forms are the ones which will be used in the next section.

## 2 THE ALGEBRA

According to Lipschutz,[14] a Boolean algebra is a set containing at least two distinct elements, 0 and 1, two binary operations, $\vee$ and *, and a unary operation, $-$, satisfying the following axioms:

A1 Commutative Laws

$$a \vee b = b \vee a \quad a*b = b*a$$

A2 Distributive Laws

$$a \vee (b*c) = (a \vee b)*(a \vee c)$$

$$a*(b \vee c) = (a*b) \vee (a*c)$$

A3 Identity Laws

$$a \vee 0 = a \quad a*1 = a$$

A4 Complement Laws

$$a \vee -a = 1 \quad a*-a = 0$$

Brown's arithmetic satisfies these laws and is in fact the two-element Boolean algebra. That there is only one two-element Boolean algebra is well known (Liu[15]).

### 2.1 *Brown's Algebraic Axioms*

Brown offers two algebraic axioms, in addition to his arithmetic principles.

(F 5) $\overline{\overline{p}\,|\,p}\,| =$

and

(F 6) $\overline{\overline{pr}\,|\,\overline{qr}}\,| = \overline{\overline{p}\,|\,\overline{q}}\,|\,r.$

We will re-write these axioms in standard Boolean notation using the second system of translation suggested in section one. (Alternatively one could use the first system and the definition of negation.) Thus, they become

(F 5*) $-(-a \vee a) = 0$

and

(F 6**) $-[-(a \vee c) \vee -(b \vee c)] =$

$$-(-a \vee -b) \vee c.$$

Using the definition of conjunction in terms of negation and disjunction,

(D 1) $a*b = -(-a \vee -b)$

(F 6**) becomes

(F 6*) $(a \vee c)*(b \vee c) = (a*b) \vee c$

(F 5*) and (F 6*) are of course basic Boolean identities; (F 5*) is a form of excluded middle, and (F 6*) is a form of the distributive laws. Thus Brown's algebraic axioms are *implied* by the axioms of Boolean algebra.

In fact, in themselves, Brown's axioms are weaker than the Boolean axioms. But as was the case with his arithmetic, Brown uses more prin-

ciples that he states. First of all, we still have with us the plethora of blanks joinable to either side of any term in any equation. More surprisingly, we find that Brown simply *presupposes* the commutativity and associativity of his concatenation operation. That is, he commutes terms and omits parentheses in his equations without ever stating as an axiom that such commutations and omissions are permissible.

Brown does not make these presuppositions unknowingly. In his discussion of Sheffer's axioms for Boolean algebra, p. 109, he says:

> Sheffer explicitly assumes the restriction of his operator to a binary scope, and also, implicitly, assumes the relevance of the order in which variables under operation appear. ... Sheffer was therefore forced to design his initial equations so ingeniously as to contradict them [the assumptions] both.

By this Brown seems to mean that Sheffer considered the possibility that his operation was not associative and not commutative, and then showed that his operation was both associative and commutative. In fact, although the Sheffer stroke is commutative, it is not associative. Thus Sheffer had good reason to consider whether these properties held for his operation.

What Brown seems to be saying is that either to assert or to deny these properties is to give them more consideration than they are due. He believes that the fundamental operations of mathematics (whatever they may be) are so necessarily commutative and associative that to *assert* that they are is to give the very possibility that they might not be more credence than it deserves. Thus he tells us that the properties are simply not "relevant". This is like objecting to another person's saying "I don't care what color a person is", because such a claim assumes the *relevance* of color, even as it denies it. The grain of truth in such a view is this: if no one ever had cared what color a person was, it *would* be odd to deny that you did, and *would* suggest that you had thought about the possibility of caring more than people who never mention it. But since many people have had a fierce interest in color, just as many mathematical systems do involve non-commutative and non-associative operations, it does not seem untoward to deny either the interest for oneself or the property to the operations.

To state commutativity and associativity is not, as Brown has it, to implicitly admit the relevance of order and binary scope; it is to explicitly point out that order and binary scope are irrelevant in the system under consideration, while allowing that they may be relevant in other systems. To treat these two properties as unmentionables is passing strange.

Some defenses for Brown's procedure do exist: one might say that Brown is proposing a logic in which the commutativity and associativity of all operations are rules of inference, so that they need not be stated as axioms of any particular system employing that logic. But even if we were to take this tack, to compare Brown's system with an ordinary mathematical system it would be necessary to add the following unstated axioms:

Associativity of Disjunction

$$(F\ 7^*) \qquad (a \vee b) \vee c = a \vee (b \vee c)$$

Commutativity of Disjunction

$$(F\ 8^*) \qquad a \vee b = b \vee a$$

Because we are treating the corner as complementation, we introduce the maximal element of the algebra through a definition

$$(D\ 2) \qquad 1 = -0.$$

But it is also necessary to note that Brown generalizes the power of his blanks to create identities beyond the type (F 3*).

First of all, from algebraic identities of the type

$$a = a$$

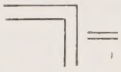
Brown's notation allows him to obtain

$$(F\ 9^*) \qquad a \vee 0 = a.$$

Again, in Brown's notation, these two equations cannot be distinguished.

Second, under the interpretation of the corner as a constant, and superimposition as an operation, the same trick ought to be possible for the superimposition operation. That is, from

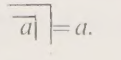
(F 2)



and

$$a = a,$$

we might obtain




$= a.$

Brown does not choose, however, to allow this kind of substitution, and instead proves the above formula as a theorem. This theorem, in our notation

$$--a=a,$$

involves a complicated sequence of algebraic manipulations. Brown's proof (pp. 28–31) can easily be reproduced from (F 7*) to (F 8*) using ordinary mathematical modes of inference.

The fact that Brown does not use the superimposition tack to establish double negation and the fact that he never superimposes variables in his algebra both provide substantial additional evidence that Brown sees the corner as an operation, and not as a constant.

## 2.2 *The Equivalence of the Two Systems*

We are now ready to show that all the axioms of Boolean algebra, (A1) through (A4), follow from (F 1*) through (F 9*). Of course, not all of (F 1*) through (F 9*) will be needed, but because all are Boolean identities, the ones not needed are merely redundant.

For reference, Brown's axioms are listed here;

(D1) $a*b=-(-a\vee -b)$

(D2) $1=-0$

(F 1*) $1\vee 1=1$

(F 2*) $-1=0$

(F 3*) $1\vee 0=0\vee 1=1$

(F 4*) $0\vee 0=0$

(F 5*) $-(-a\vee a)=0$

(F 6*) $(a\vee c)*(b\vee c)=(a*b)\vee c$

(F 7*) $(a\vee b)\vee c=a\vee(b\vee c)$

(F 8*) $a\vee b=b\vee a$

(F 9*) $a\vee 0=a.$

*Commutativity*

The commutativity of disjunction is (F 8*).

Because "*" is defined as $a*b=-(-a\vee -b)$, the commutativity of "$\vee$" guarantees that of "*".

*Identity*

The identity law for "$\vee$" is (F 9*).

For conjunction, we again use the definition, obtaining $a*1=-(-a\vee -1)$. From (D2), $1=-0$, we have $a*1=-(-a\vee --0)$, and using (F 2*), we now have $a*1=-(-a\vee 0)$, but by the identity law for "$\vee$", this means $a*1=--a$. Double negation now gives us $a*1=a$.

*Complement*

Brown's axiom (F 5*), $-(-a\vee a)=0$, gives us $--(-a\vee a)=-0$, so by double negation and the definition of "1", we have $-a\vee a=1$. Because of commutativity, this the complement law for "$\vee$".

From the definition of *, $a*-a=-(-a\vee --a)$, which equals $-(-a\vee a)$ by double negation, and by (F 5*), $a*-a$ thus equals 0.

*Distributivity*

By commutativity, $a\vee(b*c)=(b*c)\vee a$. By (F 6*) then, $a\vee(b*c)=(b\vee a)*(c\vee a)$. This last term is $(a\vee b)*(a\vee c)$ by another application of commutativity.

The second distributive law requires replacing $a$, $b$, and $c$ in (F 6*) with $-a$, $-b$, and $-c$, obtaining $(-a\vee -c)*(-b\vee -c)=(-a*-b)\vee -c$. Eliminating "*" by (D2), we have

$$-[-(-a\vee -c)\vee -(-b\vee -c)]=$$
$$-(--a\vee --b)\vee -c.$$

Negating both sides of this equation and applying double negation leads to

$$-(-a\vee -c)\vee -(-b\vee -c)=$$
$$-[-(a\vee b)\vee -c]$$

which reapplying the definition of "*" gives us

$$(a*c)\vee(b*c)=(a\vee b)*c,$$

which is the second distributive law when rearranged by commutativity.

We have now shown that (A1) through (A4) are derivable from Brown's axioms plus two

definitions. Brown's axioms, moreover, are all well known Boolean identities. The two algebras, then, are synonymous (De Bouvere[5]). They determine exactly the same class of structures. Brown and his supporters are therefore wrong when they claim he has discovered a new logic. He has reinvented traditional Boolean logic in a notation that would be a printer's nightmare, but with enough obscure and therefore profound-sounding remarks in his commentary to impress non-mathematicians. At best, Brown has produced a new axiomatization for Boolean algebra. Many such equivalent axiomatizations are known (see, for example, Mendelson, 1970) but it is very likely that a minimal subset of (F 1*) through (F 9*) would constitute a new one.

The equivalence of Brown's algebra and Boolean algebra has also been noted by Banaschewski[1] and by Gould.[6] Brown himself in his first appendix claims to prove the Sheffer axioms for Boolean algebra from his axioms. This claim implies that Brown's system is either equivalent to or is a special case of Boolean algebra, and is not a more general system as Brown seems to imply.

Despite the synonymy of the two axiom sets, one minor difference between Brown's system and Boolean algebra is that the definition of "Boolean algebra" stipulates that all Boolean algebras, in addition to satisfying the listed axioms, must have at least two elements. The true mathematical significance of this difference, we shall see, is minimal, but it looms large in saving Brown from inconsistency.

## 3 BROWN'S PARADOXICAL EQUATION

The most astounding part of Brown's work is the manner in which he tries to give meaning to relatively inconsistent equations.

### *3.1 Substitutions and Equations*

The valid equations of an algebra are the equations that follow from the axioms defining the algebra—they are the equations that are true of all structures satisfying those axioms. In a valid equation, any constant can be substituted uniformly for any variable in the equation, and the result must still be a valid equation. Invalid equations, on the other hand, may be true for some substitutions of constants for variables or true in some of the structures satisfying the axioms, but not others. Relatively inconsistent equations—relative to an axiom system—are invalid equations that are not true in the axiom system for any assignment of constants to the variables of the equation.

For example,

$$a \vee 1 = a$$

is an invalid equation which is true when "1" is substituted for "$a$", while

$$a \vee -a = 1$$

is a valid equation, and

$$a \vee -a = 0$$

is inconsistent with the definition of Boolean algebra, since from that equation and the valid one immediately before it, we have

$$1 = 0.$$

As we have noted, this result is not strictly inconsistent with Brown's axioms, but he seems to want to avoid it. He claims, on page 19, for example, that the corner and the blank have distinct values "by definition". But Brown offers no definitions of these symbols, as they are primitives of his algebra, and offers no definition of structure satisfying his system which would require this. We suspect that, in the general spirit of his work, he is confusing use and mention here (see Mates[18]).

In Chapter 11, Brown suggests a way of interpreting inconsistent equations: he allows one to substitute *different* constants for different occurrencies of the *same* variable. For example, in $a \vee -a = 0$, we could substitute "0" for the first occurrence of "$a$", and "1" for the second, obtaining $0 \vee -1 = 0$, a valid equation. But changing the rules of logic in this drastic way necessitates not only that $0 = 1$, but that there is only one object in the structure to which the logic is being applied. Otherwise, it would not be logically sound to use different constants interchangeably. In any system satisfying the axioms of Boolean algebra, as Brown's does, if $0 = 1$, there can be only one object in the system. Such a system is not mathematically exciting because *all* equations are true of it, not only those of Boolean algebra, but those of any algebra.

(Historical note: This system is not original with us or with Brown. It is the Aftermath system of Howland Owl propounded in Walt Kelly's "Pogo" comics, circa 1964.)

Brown treats an equation requiring the technique we have just discussed for its interpretation in great detail, and he feels this equation is the key to new fields of study and to the solution of old paradoxes in mathematics. This equation is

$$f=\overline{f}\rceil \tag{P1}$$

First, Brown claims that (P1) can be interpreted as

$$f_t=\overline{f_{t+1}}\rceil. \tag{P2}$$

To see again that this is nonsense, we could similarly "solve" the arithmetic inconsistency

$$x=x+1$$

by claiming it "really" means

$$(y+1)=y+1,$$

by substituting "$y+1$" for the first occurrence of "$x$", and "$y$" for the second.

### 3.2 *Applications in Automata and Switching Theory*

Even though (P1) is inconsistent, (P2) makes perfect sense. It could describe, for example, the behavior of a blinking light. Suppose the two states of the light occur for equal lengths of time. Allowing that unit of time to be assigned to "1", the equation can be read as the true claim that the state of the light at any time is the opposite of what it is after one unit of time has passed. Such a system is called synchronous. Kauffman,[11] has also discussed an asynchronous interpretation of Brown's formalism. This work of Brown's is reasonable, but it is not, as he seems to think it is, original with him (see Brown's preface).

Such systems are well known. For example, Klir,[12] describes them as discrete deterministic sequential systems, in a treatment far more detailed and sophisticated than Brown's. Of course, the idea is much older than even Klir's work. Such systems go back at least to McCulloch and Pitts.[20] Shortly after the publication of their paper von Neumann (see Randell[24]) recognized that their technique could be used to describe the logical design of a digital computer. McCulloch and Pitts had pointed out that memory could be modeled by sequences of zeros and ones circulating around a sequence of formal neurons arranged in a circle. This idea was quickly implemented. Some of the earliest digital computers had circulating memories which consisted of acoustic signals in delay circuits. These memories were later abandoned in favor of core memories that were faster and did not have to be constantly refreshed, but a faster circulation memory, the so called "bubble" memory, is currently in the developmental stage (Matick[19]). Thus when Brown claims that his equations have application to computer circuitry (page 99) he is certainly correct.

The devices Brown discusses as an extension of his time shift notation also have application in switching theory, but they are a special case of devices that have been studied and used for many years. They are usually called feedback shift registers, and were probably first described by Huffman.[9] These devices and their generalizations are so important that their study forms a standard part of the computer science and engineering curriculum. They are discussed in detail and with copious references, in Harrison and Kohavi[13] for example.

### 3.3 *Applications in Set Theory*

Brown suggests that his treatment of (P1) provides a solution to Russell's paradox, and an alternative to the theory of types (page 97). This cry is taken up forcefully in Howe and von Foerster.[8] This view is entirely mistaken.

First of all, the equation

$$f=\overline{f}\rceil \tag{P1}$$

can at best be interpreted set-theoretically as

$$B=\bar{B}$$

which is the claim that the set $B$ is equal to its own complement, or

$$B=\{x|x\notin B\}. \tag{S1}$$

This is to say that $B$ is the set of all objects that are not members of $B$.

But (S1) has nothing in particular to do with Russell's paradox. Russell's paradox concerns the purported set $R$ which contains all the sets that are not members of *themselves*, i.e.,

(S2) $$R = \{x | x \notin x\}.$$

The Russell set, defined by (S2), and Brown's set, described in (S1), are not described in comparable equations. (S2) is paradoxical precisely because it *looks* like a good definition. (S1) is not a good definition, as the term to be defined, $B$, occurs on both sides of the equation.

Russell's paradox is the proof that if $R$ existed, it would both contain and not contain itself, and hence would force us to accept an inconsistency. Nowhere in that proof does one find a claim that corresponds to (S1).

Indeed, in one of the common solutions to Russell's paradox for set theory, namely the von Neumann[28] distinction between sets and classes, the Russell set *does* exist as the universal class, but in no possible set theory could the "Brown set" exist. Investigating (S1), we find it immediately implies that $x$ is in $B$ just in case $x$ is not in $B$, from which it follows that everything would both be *and* not be in $B$. Thus, $B$ would be *both* the universal class and the empty set. It appears we have only returned to a one object system, but by allowing the relation of membership into the language, we are worse off than when dealing with Brown's purely equational language. $B$ itself can still not exist, for *it* would both be and not be a member of itself. Hence, (S1) can be true only in "set theories" in which there are *no* objects.

An important point is being presumed here—Brown's algebra is a system of *equations*, and as such is expressed in an extremely *weak* language, in which only small portions of mathematics could *possibly* be expressed. As a system of equations, Brown's algebra lacks quantifiers, external or genuine negation, and even relational symbols. Almost all statements in even so elementary a branch of mathematics as set theory require these devices, and in particular, every formula involved in Russell's proof employs such symbolism and could not, therefore, even be *expressed* in the language of *Laws of Form*. These remarks also apply to Varela's extension of Brown's system.[26,27]

It is the poverty of equational languages which saves Brown from absolute inconsistency. To see this, first note that in saying Brown's language lacks a genuine denial operator, we are pointing out that his corner operates on variables and constants *in* his language, while genuine denial operates on the complete statements *of* a language itself. For example, in Boolean algebra, the internal complement operator "$-$", represents a function on objects of the algebra, as in

$$-0 = 1,$$

but a genuine negation operation added to such a language would allow us to say things like

"$0 \neq 1$" (or "not $[0 = 1]$").

This distinction may be clearer in numerical algebra, for here the first equation is nonsense, and the second formula true. Note that absolute inconsistency involves asserting and denying simultaneously a single proposition, or claiming that one statement, given a precise single interpretation, is both true and false. Now, without external negation in a language, there is no way to assert that any of its statements are false, and hence no way to arrive at an absolute inconsistency.

### *3.4 Paradox and Rationality*

There are three kinds of "paradoxes": mathematical-scientific paradoxes, social paradoxes, and inconsistencies. Mathematical-scientific paradoxes are puzzling "paradoxical" *truths* that are paradoxical in that they are surprising; and they surprise because there are other truths that *appear* to be inconsistent with them, or because there are other false beliefs we wish to maintain that *are* inconsistent with the paradoxical truth. Russell's paradox, that there exists no Russell set, is such a paradox in that it is or appears to be inconsistent with the abstraction principle that every concept determines a set. It requires a solution in that we must *now* either explain *why* the abstraction principle, which is quite attractive, is not true, or why the abstraction principle, when stated correctly, is not *really* inconsistent with Russell's result. Such paradoxes are a major source of new directions in the sciences.

We have argued, however, that Brown does not offer such a solution and new direction, since he does not even confront that paradox. He and

his supporters, on the other hand (see Brown's preface, and Howe and von Foerster[8]), suggest that Brown offers the first, one, and only such solution, thus opening up a new field of research and a new way of understanding mathematics and systems in general. We wish to point out that if people *are* troubled by Russell's paradox, the development of alternative solutions to it and the corresponding new logics and languages required in those solutions have by now a long and rich history. (One may consult, for instance, Ramsey,[23] Schilpp,[25] or Martin,[17] to get a sampling of such discussions and more extensive bibliography.)

Similar to this first kind of paradox in mathematics and science are social paradoxes of which Marxists and Hegelians are often especially fond —for example, the disturbing and unacceptable juxtaposition of opulence and squalor. Such paradoxes provide a source of new visions, energy, and creativity, just as the mathematical-scientific paradoxes do. But both of these kinds of paradoxes are often confused with the third kind of paradox: antinomy or absolute inconsistency. Absolute inconsistency is intolerable in *any rational*, or simply any *language-using* system, for it involves the absolute breakdown of even the possibility of communication. If a statement that things are a certain way does not rule out the possibility that they are *not* that very way, the statement can communicate no information. If you tell me you are bald, and I accept inconsistencies, even if I believe you I will not yet have any definite beliefs, for I may still believe that at the very same time in no sense at all are you bald. Now, it is precisely because this third kind of paradox is intolerable that the other kinds are the source of so much energy. The energy is expended to avoid absolute inconsistency.

Howe and von Foerster, along with many others who see consistency as a bugaboo, confuse these kinds of paradox. In fact, they talk about paradox, inconsistency, self-reference, and the relationship of observer to observed as if all these things were indistinguishable. There is, in fact, no reason whatsoever to see the theory of types as standing in *any* relation, either supportive or contrary, to the existence of constructive "paradoxical" relationships in psychiatry. (For a discussion of what the theory of types *does* entail, see Chihara[3]).

General Systems Theory, if it is to be general, must admittedly involve the discovery of similarities between strikingly different kinds of systems. If it is to be a science, however, and not a branch of magic, the features different systems share must be exactly specified; that is, the scientific way to generality is through equivalence in which distinct entities are considered to be the same for given purposes through their participating in a well-defined equivalence relation. In this regard Brown's *Laws of Form*, and its supporters, do a disservice to systems theory as a science.

## CONCLUSION

We have shown that Brown's axiom system specifies exactly the Boolean algebras. Thus Brown's system has applications to the extent that Boolean algebra has applications. In particular, Boolean algebra has long been used in the design and analysis of computer and communication systems, so that Brown is suggesting nothing new when he claims that his system has applications in these fields.

Boolean algebra is only a small fragment of logic. As such it does not contain either quantifiers or membership, and thus it is impossible to even state Russell's paradox within Boolean algebra. Since Brown's system is synonymous with Boolean algebra, it suffers from the same deficiencies. What is claimed to be a solution of Russell's paradox is based on misunderstanding and confusion of terms.

While we recognize the power of paradox to stimulate thinking, we feel that the solution of paradoxes and the clearing up of confusion is progress in science. If General Systems Theory is to become a science, we must avoid confusions without destroying useful analogies.

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# AUTOMATION OR AUTONOMY IN ORGANIZATIONAL DESIGN

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"*...For the first time in History, man is obliged to choose freely his future and the future of mankind and he is obliged to do so by the very fact that he has, for the first time, the freedom hence the responsibility of doing so*".

*Denis de ROUGEMONT*

Today there is growing criticism of the Input/Output (I/O) model as a conceptual tool for explaining and understanding how systems are functioning. These models have been particularly useful—mainly in engineering, where simple, static and open systems are involved—but they are, however, unable to explain the functioning of living and social systems. This paper will present evidence, from various fields, illustrating the relevance of autopoiesis. Furthermore, it will relate the concepts of autonomy and identity to the way in which various social systems function within the environment.

INDEX TERMS: Input/output systems, autonomy, identity, autopoiesis, morphogenesis.

## INTRODUCTION

Many physical phenomena can be explained by precise relationships, often called physical laws, which express cause-and-effect relations between, or among, a number of factors. Boyle's law, for instance, states the relationship between volume, $v$, and temperature, $p$. The exact mathematical law is:

$$p = \theta \frac{N}{v} \qquad (1)$$

where $N$ is the number of modecules and $\theta$ is a proportionality constant.

The causality postulated by (1) is obvious: if the volume, $v$, changes—all other things being held constant—pressure also changes, but inversely. Furthermore, it is not difficult to measure the extent of this influence precisely, by using (1) above in a way that could be useful for a variety of applications.

Engineers have carried the idea of causal models a step further by introducing the notion of control as a guiding principle of the system's behaviour. They can, therefore, build systems which could achieve, or maintain, a specific task, i.e. output, by manipulating the input(s) appropriately. This input/output (I/O) control system, made explicit by cybernetics,[1] has gained widespread popularity and has greatly contributed to the automation of a large variety of tasks.

The I/O control model of designing and building systems has also influenced the thinking of many outside the field of engineering. Econometricians, for example, have attempted to discover a causal relationship between the level of economic activity and the numerous factors that determine it;[2,3] organizational theorists have tried to explain various concepts through formal cause-and-effect models;[4,5] management scientists have built a great number of mathematical models relating inputs to outputs.[6,7] However, the most extreme form of the I/O idea is found in

behaviouristic psychology[8,9] which advocates a simple stimulus-response ("S-R", i.e. I/O) model to explain human behaviour. Although most formal developments in I/O systems theory deal with ways of better representation of the internal states of the systems,[10] this aspect has not been disseminated outside of the systems engineering field. From the point of view at least popular amongst social scientists, I/O systems attach little importance to, and requires virtually no insight into, the system or process it is applied to; the system is basically viewed as a "black box". The question that must be asked, however, is whether or not such a simplistic model can adequately explain high level systems[11] such as social or living ones—by itself. It is believed that it is not appropriate for anything more complex than mechanistic, engineering-type operations, with a few variables and constant or preductable inputs. (See Exhibit 1). The main point of this paper is not to convince the reader of the importance of what have been called autopoietic organizations, as to this end many eloquent literature sources exist.[12,13,14,15] Rather, the paper attempts to enlarge the applications of the concept of autopoiesis to areas other than biology. This will be done by discussing several illustrations and providing many examples of applications of the concept in a variety of fields of social sciences—and in particular, that of management. Secondly, it shows that a number of theorists can be cited,[18-21,26,27] who have been convincingly providing autopoietic arguments, even though they were not actually calling them by such a name.

## A BASIC ANTINOMY: HIERARCHICALLY CONTROLLED VERSUS AUTONOMOUS SYSTEMS

Disenchantment with I/O thinking is not completely new or limited to single disciplines. Several theorists in the field of social sciences can be cited,[18,19] who have convincingly argued against the I/O approach and who have proposed alternative approaches with far-reaching explanatory powers.[20,21] As Zeleny[22] has pointed out, autopoiesis has been emerging as a new paradigm of thought for more than a century. However, most of the impetus has come from Maturana and Varela.[23,24] Their thinking and research has developed mainly from biology; their striking example of why an I/O model is inadequate to explain reality is taken from the effects of immunization.[25]

Certain living systems somehow have the ability to *recognize* a foreign substance. This has been shown by some biologists by the capacity of the organism to generate, when necessary, a specific antibody to counteract each specific invader. This presupposes both the capacity to identify each foreign substance *and* a large reserve of antibodies to select from. This is indeed a typical I/O mode of operation, whereby the organism, through experience and learning, gradually builds up a reserve of responses to deal with incoming stimuli adequately. However, this kind of model becomes extremely complicated, if not unable, to explain how new synthetic substances, which did not exist before, and therefore could not have been encountered and experi-

EXHIBIT 1

| Range and/or type of Inputs | Number of Inputs: A FEW | LARGE | INFINITE |
|---|---|---|---|
| CONSTANT | I/O system is adequate | I/O system is possible | I/O system impossible |
| CHANGING BUT PREDICTABLE | I/O system can work with external modifications | I/O system extremely difficult or impossible | I/O system impossible |
| CHANGING BUT UNPREDICTABLE | I/O system cannot work without constant outside supervision | I/O system impossible | I/O system impossible |

enced in the past, are nonetheless recognized and neutralized. Varela proposes an elegant solution by shifting the emphasis from the process of reaction to every foreign substance (input) by neutralization (output), to the *process of recognition* that a certain material is indeed a foreign substance and *not* part of oneself. Equally important is the *existence* of a process, ability or function, which can generate an infinite number of antibodies and which is *flexible enough to deal with any and all possibilities.* This is very different from an I/O model, or a trial-and-error mode of operation.

Another similar example can be found in Festinger's work[26] on consciousness. Using several elegant experiments, he argued for an alternative theory to the prevailing I/O model of vision. A brief description of an experiment he carried out will further illustrate the inadequacy of the I/O approach. A subject is given two prismatic contact lenses to wear on his eyes, and the experimenter gives him a straight line to look at. His vision at first tells him that this is a curved line. This is indeed a true impression, consistent with the curvature on his retinal image. However, as he keeps looking at the line, moving his eyes from one end to the other, he sees less and less curvature and after some minutes he in fact sees a straight line. Festinger explains that while the subject is attempting to direct his eyes so as to continuously focus on the line, the muscles of his eyes move in accordance with the fact that the line is straight in spite of his subjective experience of curvature, determined by the prismatic lens. Thus, as the eye muscles are instructed to move for a straight line, the conscious perception shapes up according to this fact and after a while the line is seen straight. Therefore, the conscious perception of the line is not the organization of informational inputs from the eyes, but the organization of *efferent* signals to those eyes; what the subjects see is what they actually do with their eyes.

Another convincing case of the inadequacy of I/O theories can be found in the explanation of linguistic performance. In a well-known article, Noam Chomsky wrote a definite rebuttal to the claim advanced by behaviourists to account for "verbal behaviour" by "S-R" (Stimulus-Response) type models:

> It appears that we recognize a new item as a sentence not because it matches some familiar item in any simple way but because it is generated by the grammar that each individual has somehow and in some form internalized, and we understand a new sentence, in part, because we are somehow capable of determining the process by which this sentence is derived in this grammar.[27]

A paradigmatic shift in linguistics has resulted from Chomsky's theory, precisely by moving away from an externally caused model of behaviour towards a theory of the internal competence of the speaker. In other words, a speaker possesses a certain creative capacity which enables him to reproduce or understand new sentences which he had *never* uttered or heard before. This is something which cannot be accounted for by a mechanistic, behaviouristic I/O type model.

All these examples suggest that living organisms are able to develop an "internal milieu", and can therefore maintain relative stability inside the system boundaries, in spite of external turbulence. This distinctive feature of the living systems, as Canguilhem has shown, is a capacity for generating internal norms:

> ... life being not only submission to the environment but institution of its own milieu, sets thereby values not in the environment, but in the organism itself.[28]

Another illustration of the importance of autonomy is given indirectly by Wilden in a stimulating book, where he discusses the contributions of authors such as Bateson, Chomsky, Spencer-Brown, Lacan, but not Varela, in their attempt to go beyond the limitations of I/O models towards a systems approach:

> ... it is almost universally accepted that the behaviour of any system, open or closed, informational or energetic, organic or inorganic, is a function of the way the observer-participator *punctuates* it.[29]

But Wilden still stresses the hierarchical nature of organizations (in the form of the Russel–Whitehead Theory of Types) and their quality as open systems, while Maturana and Varela point out the necessity of looking at living and social systems as organizationally *closed systems,*[30] thus stressing the need for self-referential description.

> ... wholes, or whole systems, are arranged like Chinese boxes. One whole contains another whole and every whole is contained in another whole. There is a recursion principle there. But that does not mean that you cannot stop at some point and consider a system. And I have come to the conviction that the key to understanding the wholism of such system, the wholeness of systems, is to understand that they are organized, their parts are organized, in a circular form. That is, every part interacts with every other part. That gives us a total self-referential system.[31]

Maturana and Varela's essential point is not only that autonomy is an important concept but also that self-reference is a necessary process for understanding the essential emerging properties of living systems, and consequently it is necessary to look at such living systems as closed systems. Autopoiesis is a characteristic and consequence of autonomy and self-reference. It is a process of creating oneself. Varela's views are valid for all living systems but take particular significance for social systems where the symbolic processes are so central and where communication is not only the transmission of information but also the elaboration of meaningful social codes through dialoguing autonomous entities.

In the rest of the section, several other examples will be taken from four levels of organizational complexity which illustrate further the concept of autonomy and the antinomy between autopoietic and I/O type systems.

*Workers: Robots or Agents?*

"Scientific management", a school of thought founded by the engineer F. Taylor,[32] advocated the introduction of an engineering oriented approach to the management of operations at shop-floor level. Through time-and-motion studies, and the separation of conception from execution of work, various tasks were designed in such a way as to maximize efficiency and productivity. A worker was not expected to think, but simply to execute movements as instructed. The result was the mechanization of different tasks, and the downgrading of the human worker to the level of an automaton. This dehumanization of work led to some unrest in the work force, as boredom became increasingly unbearable. Various remedies, such as "job enrichment" and "job enlargement" of the tasks performed were introduced with mixed results, but without basically changing the logics of the approach. As an alternative to Taylorism, the socio-technical school of thought advocated autonomous work groups as the basis of work organization.

Originally, the socio-technical school was initiated in England through the joint efforts of a coal miners' union and the Tavistock Institute of Human Relations, and this trend, known as "industrial democracy', spread largely to the Scandinavian countries.[33,34] The basic philosophy of this new approach is to use the distinctive resources of human agents as fully as possible: namely their willingness for initiative and their capacity for membership and self-regulation. Groups of workers are given the responsibility of organizing themselves as they like, in order to accomplish a certain task. Not only do they organize themselves, but as a result, they also tend to request more and more responsibility; for instance, direct contact with customers to follow up their defective products, re-organization of working procedures and methods, processing of technical or commercial information, setting of goals and objectives, etc. By so doing, these autonomous groups tend to gradually take over some of the responsibilities formerly restricted to various functional staff services.

*Control: Check or Self-Regulation?*

As an organization grows, various organizational changes become necessary; for instance, moving from a functional to a divisional type of structure. This entails decentralization and delegation of responsibilities, with the corresponding authority, to lower levels of executives who can manage their units with a larger degree of autonomy. Correspondingly, it was felt necessary to introduce what has been known as "management control systems"[35] which could provide proper information as to the actual achievements of various departments as compared with their objectives, the purpose of which is to be able to detect deviations early enough so as to take corrective actions rapidly when necessary.

The philosophy of "management control systems" was developed with the necessity of entrusting lower echelons within the organization with larger degrees of autonomy. This developed rather well within Anglo-Saxon countries, but proved to be somewhat more difficult to implement within Latin countries, as many multinational companies have experienced. Students[36,37,38] of control systems in Latin countries suspect the main reason to be the reluctance among Latin managers to trust their subordinates enough to really delegate responsibilities and authority to them. The result of this attitude is that the "management control system", applied in Anglo-Saxon cultures where it is tacitly understood, accepted and encouraged that everyone strives for more autonomy, finds itself in jeopardy in Latin countries.[39] In the latter, not only does this tacit assumption not hold, but worse, the reverse seems to be the case, namely

that there is enough mistrust among managers and subordinates.

The same type of antinomy that exists on the shop-floor is found here too. Among Latin cultures, control is understood as a *checking* device that a superior maintains over his subordinates, which is the engineering sense of the idea of control. On the other hand, among Anglo-Saxons, control is understood as a *regulating* mechanism enabling both superiors and subordinates to increase their responsibility and be given more autonomy.

*Corporate Strategy: Conglomerate or Synergetic?*

Conglomerates are made up of a number of business firms, which are not necessarily related, under a single top management. They flourished in the 1960s when the idea was to grow through the acquisition of any business opportunity likely to generate positive cash flow if properly managed. Synergetic corporations, on the other hand, can be as large and complex as conglomerates, except that growth is not attempted unless what is to be added will contribute to the distinctive competence of the existing organization. In this respect, growth is the result of either vertical or horizontal integration in carefully selected directions. These two kinds of organization (conglomerate and synergetic) have opposite senses of corporate identity. Conglomerates are only interested in growth for the sake of size and because it is believed that enough positive cash flows can be generated to make the entire organization more profitable. Their sense of corporate identity is low, if it were to exist at all, and it would be of a rather kaleidoscopic nature. Their only objective, which is imposed from above, is high short-term profitability. Synergetic organizations, on the other hand, possess a strong corporate identity and profitability, even though important, is not an end in itself, but a means to implement the corporate strategy. A careful study of these two extreme corporate philosophies reveals a great deal about the concepts of autonomy and identity, to be examined more fully in the next section. It should be noted, however, that very few conglomerate corporations have been consistently successful even though they were extremely popular in the 1960s, thereby demonstrating a lower capacity to adapt to changing environmental consitions.

*Centralized Nation-State versus Autonomous Regions.*

In the same way that corporate strategy has to include the participation of members of the organization as far down as possible, in order to be correctly formulated and successfully implemented, so the establishment of national policy has to be the combined effort of regional contributions. Denis de Rougemont argues very convincingly that a challenge for modern industrialized nation-states is their capacity for allowing regional participation in public management. Rougemont pleads for the development of an important kind of power:

> the power one takes over himself because it means freedom but also responsibility.[40]

and stresses the necessity "of the accession to power over self not power over others". One cannot think of more convincing plea for autonomy than in the political domain, which should go beyond national affairs to the international sphere.

*Hierarchical Coordination and Identity*

The illustrations considered in the last section show the existence of a wide range of organizational structures and processes, and a variety of concepts of how the universe can be explained and dealt with. Each organization has strengths and weaknesses. It must be viewed in relation to the task it has to achieve and the environment within which it exists. In autonomous subsystems the various units/members are more motivated. They are "masters" of their own fate by being allowed to communicate with their environment. Goal determination is done by consensus rather than being imposed from above. Various constraints are explicitly acknowledged when setting goals and during action. Coordination among the autonomous subsystems can be more difficult, and effective communication takes much longer, since there are many more links in decentralized structures. But at the same time, each unit is part of the overall decision-making process of the system. Changes in the environment can, therefore, be known much more easily with autonomous groups; furthermore, they can be dealt with faster than when changes are imposed from above. Learning, self-organization and adaptation can be achieved more easily because they are mainly left to the operating level to initiate and

implement rather than being decided on at the highest level and then being pushed downwards. In final analysis, social systems are better managed by consensus rather than coercion and this is precisely where the benefits from autonomous systems are derived.

I/O type systems have limited links with the environment, which must be regulated centrally, otherwise the system can become unstable. This presupposes, however, two things: first that the environment can be simplified enough so that all its interactions with the system can be modeled into a limited number of variables (usually few); second, that the environment will remain constant in such a way that the variables identified as the most critical will still remain the same; moreover, their range must not go beyond limits which can be regulated.[41] In the case of important permanent changes in the environment, the only alternative is to have these changes identified from above and at the same time possess an efficient procedure for re-organizing those functions of the system that need to be modified. In other words, an I/O system cannot learn, self-organize itself or adapt to environmental changes. Its major characteristic is simplicity, ease of communication, efficiency in operating and effective reaction to a specified number of outside responses, as long as they remain constant and within a certain range. Thus, the very strengths of an I/O system exclude learning, flexibility, complexity, larger number of possible responses and the ability to scan the environment generally. Furthermore, it cannot self-organize itself or adapt. These are tasks that can only be achieved by systems which possess, as "mind" does, the above-mentioned characteristics.

*Autonomy and Identity*

If there are advantages in autonomous subsystems, then why shouldn't the idea be carried to its extreme—complete independence of each subsystem from the rest? Autonomy, however, does not mean separation, and paradoxically there is less change of separation taking place in decentralized than in centralized type systems. Centralized operations are tightly controlled. People working in such organizations are given little initiative and they must carry out orders from above as closely as possible. In the extreme case of centralized structures, as in Taylorism, humans are viewed as machine processors in a chain of input/output operatiins. They stay together as a system because of strong supervision, coupled with adequate rewards and punishments. The sense of organizational identity is low and limited to a few top levels in the organizational pyramid. Thus, if the chance of separation arises, it will be taken up happily. Lack of organizational identity can also be seen in Latin countries where employees and workers are tightly supervised. In such countries, identity comes from loyalty to subsystems, for instance a union, usually in constant conflict with management and among themselves because of the lack of common objectives.

In autonomous groups, even though there is little or no control, there is a higher sense of organizational identity, a result of the very nature of autonomy. Each worker or employee is part of deciding on the overall organizational goal and the specific objectives that must be achieved. This means that he or she understands the need and advantages of being part of a larger organization and is aware of the goals and objectives. Conflicts are solved by dialogue, rather than bargaining, between subsystems which share some supra-ordinate goals.

Strong national identity can similarly be the result of greater regional autonomy. This is even more true when heterogeneous populations make up a nation. History is full of instances where tight controls or oppression have not worked as a measure to achieve national unity. On the contrary, it has provided the driving force for regional struggle, or revolution, and eventual independence. On the other hand, growing regional autonomy can strengthen national unity and result in countries of people with a high sense of national identity. The same way that oppression does not result in national unity, separation of a region from a nation creates as many difficulties and problems for the newly independent region as it solves. History is again full of examples of this kind, in which separated regions were worse off on their own than when they belonged to a bigger nation.

*Conclusions*

Input/output type systems have only a very restricted range of applicability where organizational social systems are concerned. Even when standard tasks are considered and when the environment is constant I/O modes of operation

are inappropriate as long as people are involved. Theoretically speaking, an I/O mode of operation should be more efficient and productive, but to be effective, it requires tight controls which were difficult to apply historically, and which are even more difficult in our times of high social awareness. The results of humans being obliged to behave like machines is that they get bored, the quality of what they do diminishes, and they change jobs often. Furthermore, they create strong unions which obtain enough benefits as to diminish the advantages of the I/O approach such as those originally perceived and implemented by Taylor. It is not an accident that today there is no advocate of "scientific management" nor that the psychological and social problems of managing people are in the foreground. Participative management[42] and what has been known as industrial democracy[43] are clear illustrations of the direction that management and work organization is evolving in.

When intricate interactions within a complex social environment are involved, the I/O approach is useless. The mistake which is often made, however, is that this is not so[44] and that complex interactions of a system with its environment can be modeled, and causal relationships—involving only important variables—can be found. This belief grew strongly in the sixties among westerners, when large computers first appeared and when mathematical models were thought to be able to solve all problems facing mankind.[45] The seventies have tuned down these hopes. Ecological considerations, quality of life concerns, social awareness and the understanding of the severe limitations in both sciences and human rationality have made man much more aware of his limitations, and increased the need for the concept of autonomous systems. The lesson is that the environment is too complex to be formally modeled. Instead, a constant dialogue is needed between man and man, and between man and the environment. This might mean that boundaries will need redefining, changes in the interaction pattern might be necessary, or structural changes should take place.[46] The implications for management and organizational theory is that the capacity of social systems for self-reference has to be stressed, namely the capacity for reflection, the capacity for defining one's own boundaries and sense of belonging.

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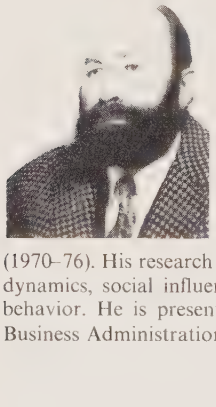

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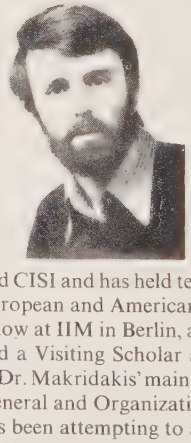

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# ON THE MEASURE OF FUZZINESS AND NEGATION
## Part I: Membership in the Unit Interval

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We investigate the meaning of the concept of fuzziness. It is suggested that fuzziness can be related to the lack of distinction between a set and its negation. Using fuzzy subsets with grades of membership in the unit interval, measures of fuzziness are suggested. First, a class of measures based on metric distances is used to derive numeric measures of fuzziness of a fuzzy set. Finally, the concept of compatibility is used to develop linguistic measures of fuzziness.

INDEX TERMS: Fuzzy set, measure of fuzziness, negation, linguistic measure.

## INTRODUCTION

Fuzzy sets are playing a significant role in the expanding field of general systems research .[1,2] In part, their usefulness is due to their ability to model systems which involve imprecise components and connections,[3] a characteristic of many biological and social systems. Fuzzy sets also provide a medium in which it is possible to represent and simulate the performance of human experts at complex tasks.[4] In a recent publication in this journal, Tong[5] has used fuzzy sets to synthesize models for industrial processes. In developing a general theory of planning, Linde and Goguen[6] find fuzzy sets a useful framework in which to discuss some of their ideas. Bona and Villa[7] use fuzzy sets to help construct a general theory for feedback control of uncertain systems. In discussing robotics, Goguen[8] shows the advantage of including fuzziness in the design of robots as an aid in providing systems that are robust and adaptive.

The question naturally arises as to what is the essence of fuzziness and how can it be measured. It is the purpose of this paper to try to present, in addition to measures of fuzziness, an intuitive understanding of the meaning of fuzziness. By being able to understand and measure fuzziness in systems we can use these measures to help include and measure robustness and adaptability in systems as suggested by Goguen.[8]

The ability to measure fuzziness in a system gives us the facility to compare different systems along this parameter. It enables us, when studying hierarchies of systems, to see to what degree fuzziness is a property of increasing complexity.

The concept of entropy plays a crucial role in measuring information in systems having probabilistic uncertainty. It is envisioned that a measure of fuzziness, as presented in this paper, would play a similar role in systems involving possibilistic uncertainty.

Furthermore, a considerable amount of literature in general systems is concerned with defining concepts associated with systems and their interrelationships.[9] Zadeh[10] has suggested that many complex ideas, such as those appearing in general systems theory, can be defined via fuzzy sets. The ability to measure the fuzziness of concepts will enable one to characterize and compare many definitions as to their precision or lack of, their adaptability to future extensions and their usefulness in suggesting new creative ideas and relations.

Since Zadeh[11] first introduced the concept of a fuzzy set various authors have attempted to define measures of fuzziness of a fuzzy set.[12–19] For the most part, the exceptions being Pollatscheck[16] and Kaufman,[19] these articles have been influenced by the initial attempt of DeLuca and Termini,[12] which itself was influenced by Shannon's measure of entropy.

DeLuca and Termini[12] state three conditions which they feel a measure of fuzziness should satisfy. These conditions state that if $f$ is a fuzzy subset over the set $X$, which takes membership

grades in the unit interval, then if $d(f)$ is a measure of the fuzziness of the subset $f$ then $d(f)$ should satisfy:

$P_1$: $d(f)=0$ if $f$ is a crisp set, that is if $f(x)\in\{0,1\}$ for all $x\in X$.

$P_2$: $d(f)$ should attain its maximum value for the fuzzy subset $f(x)=\frac{1}{2}$ for all $x\in X$.

$P_3$: $d(f)\geqq d(f^*)$ if $f^*$ is a sharpened version of $f$. That is, if $f^*$ is a fuzzy subset such that $f^*(x)\geqq f(x)$ if $f(x)\geqq\frac{1}{2}$ and $f^*(x)\leqq f(x)$ if $f(x)\leqq\frac{1}{2}$.

De Luca and Termini then proceed, motivated by Shannon's concept of entropy, to show that

$$d(f)=-K\sum_{i=1}^{n} f(x_i)\ln f(x_i) + (1-f(x_i))\ln(1-f(x_i))$$

is a function which satisfies these conditions. Knopfmacher[15] and Loo[18] subsequently showed that under conditions $P1$–$P3$ the selection of $d(f)$ is not unique but that there exists a large class of functions that are admissable as measures of fuzziness.

We feel that this approach to fuzziness, though useful for obtaining measures of fuzziness, doesn't capture the essential nature of fuzziness. First, we feel that the requirement of trying to obtain $d(f)$ as a number may distort the essential idea of fuzziness. Secondly, we feel that simply stating conditions which a measure of fuzziness must satisfy fragments the concept of fuzziness without giving it a holistic meaning. That is, one gets lost in the mathematical conditions specifying $d(f)$ without getting to see what it is $d(f)$ is measuring. Thirdly, being confined to obtain a measure of fuzziness that is a real number limits the ability of these approaches to deal with fuzzy subsets whose membership grades lie in lattices.

In this paper we present an alternative approach to the concept of measuring fuzziness which we feel leads to a more intuitive understanding of the meaning of fuzziness. We also try to develop, based upon our idea of fuzziness, some measures of the fuzziness associated with a fuzzy subset. We find that in the cases where the approaches taken by the other authors are applicable, that is where membership grades lie in the unit interval and positive real numbers are desired to measure $d(f)$, our results are the same as our predecessors. However, we are able to handle situations in which the measure of fuzziness itself is a fuzzy subset of unit interval. Furthermore we are able to handle fuzzy subsets whose grades lie in certain lattices.

## FUZZINESS AND NEGATION

In order to develop measures of fuzziness for a fuzzy subset we must first isolate the concept and the location of fuzziness in set theory. Ordinary set theory consists of a set $X$, a set of objects called subsets of set $X$, the two binary operations, defined on these subsets, called union and intersection, a binary operation called negation and two specially indicated sets called $X$ and $\Phi$.

These sets along with these operations form a *Boolean algebra*. That is,

1) union and intersection are commutative.
2) union and intersection are associative.
3) union and intersection are idempotent.
4) union and intersection are absorptive.
5) union and intersection are distributive.
6) $X$ and $\Phi$ are respectively maximal and minimal elements of the algebra.
7) negation satisfies the property of being complementary that is, $A\cup\bar{A}=X$ and $A\cap\bar{A}=\Phi$.

Zadeh[11] defines a fuzzy subset of $X$ as a subset of $X$ in which membership grades lie in the unit interval. In addition, he defines $A\cup B=C$ such that $C(x)=\text{Max}\,[A(x),B(x)]$, $A\cap B=D$ such that $D(x)=\text{Min}\,[A(x),B(x)]$ and $\bar{A}$ by $A(x)=1-A(x)$. It can quite easily shown[19] that the fuzzy subsets of Zadeh satisfy the first six of these conditions while not satisfying the seventh condition. That is, for fuzzy sets

$$A\cup\bar{A}\neq X$$

$$\text{and } A\cap\bar{A}\neq\Phi$$

That is, fuzzy subsets don't satisfy the law of the excluded middle and the law of contradiction.

Furthermore, we note that the first six axioms of a Boolean Algebra do not involve the negation. This implies that any property of a Boolean Algebra which can be derived from the first six axioms is valid for fuzzy subsets. In particular, the notion of a partial ordering developed by the rule $A\leqq B$ or $(A\subset B)$ if $A\cap B=A$ or equivalently $A\cup B=B$ is definable on fuzzy and crisp sets. This idea introduces the concept of sets being contained in another set.

However, for our purposes the significant fact is that the only distinction between fuzzy and

ordinary sets lies in the non-satisfaction of the complementary axiom, which in turn is the only axiom involving the negation of sets subsets. Thus, it appears that the essence of fuzziness lies in the relationship between $A$ and $\bar{A}$.

While fuzzy subsets don't satisfy complementarity they do satisfy two other properties involving negation derivable from the Boolean axioms.[19] These properties are,

(1) $(\bar{\bar{A}}) = A$ (involution),

(2) $(\overline{A \cup B}) = (\bar{A} \cap \bar{B})$

$(\overline{A \cap B}) = (\bar{A} \cup \bar{B})$ (De Morgan's laws).

That is, both Zadeh's fuzzy subsets and the ordinary subsets satisfy involution and De Morgan's laws.

Furthermore, these two properties imply the following:

PROPOSITION 1. *If* $A \subset B$ *then* $\bar{B} \subset \bar{A}$

*Proof:* $A \subset B$ implies
$A \cup B = B$ (definition of containment);
$\bar{A} \cap \bar{B} = \bar{B}$ (De Morgan's law);
$\bar{B} \subset \bar{A}$ (definition of containment).

This implies that negation is order reversing and also as $A$ increases $\bar{A}$ decreases.

PROPOSITION 2. $A \cap \bar{A} = \bar{F}$ *iff* $A \cup \bar{A} = F$

*Proof:* If $A \cap \bar{A} = \bar{F}$ then $\bar{A} \cup A = F$ (De Morgan's law), and then $\bar{F} = A \cup \bar{A}$ (commutativity). Similarly the opposite way.

From properties one and two we note that as the intersection between $A$ and $\bar{A}$ increases the union decreases and vice versa.

PROPOSITION 3. $\bar{\Phi} = X$ *and* $\bar{X} = \Phi$

*Proof:* $\Phi \cup \bar{X} = \bar{X}$ (definition of $\Phi$);
$\bar{\Phi} \cap (\bar{\bar{X}}) = (\bar{\bar{X}})$ (De Morgan's law);
$\bar{\Phi} \cap X = X$ (involution), thus implying $\Phi \supset X$, which in turn implies $X \cup \bar{\Phi} = \bar{\Phi}$, but the definition of $X$ is that $X \cup A = X$; therefore $\bar{\Phi} = X$.

PROPOSITION 4. *If* $A$ *is a fuzzy set then*

(1) $A \cap \bar{A} = \Phi$ *iff* $A \cup X$, *and*

(2) $A \cap \bar{A} = \Phi$ *iff* $A$ *is a crisp set.*

*Proof:* (1) If $A \cap \bar{A} = \Phi$, then $A \cup \bar{A} = \bar{\Phi} = X$ and, similarly, If $A \cup A = X$, then $A \cap A = \Phi$. (2) If $A \cap \bar{A} = \Phi$, then Min $[A(x), (1 - A(x)] = 0$ for all $x \in A$; then either $A(x) = 0$ or $1 - A(x) = 0$ for all $x$ and, therefore, $A$ is crisp. A crisp obviously implies $A \cap \bar{A} = \Phi$.

With these observations in mind we can be more specific in our search for a measure of fuzziness of a fuzzy subset $A$ of $X$. We noted that what was missing from fuzzy sets was the complementarity property. We note that complementarity means that $A$ and $\bar{A}$ are completely distinct and completely exhaustive. We also noted that complementarity implies De Morgan's laws and involution, two properties which fuzzy subsets have. Furthermore, the above propositions indicated that involution and De Morgan's laws imply that the more distinct $A$ and $\bar{A}$, the smaller $A \cap \bar{A}$, the more exhaustive $A$ and $\bar{A}$, the larger $A \cup \bar{A}$. So that, the satisfaction of involution and De Morgan's law by the negation operation implies a connection between distinction and exhaustiveness by a set and its negation. That is, the satisfaction of one leads to the satisfaction of the other. This duality between these concepts allows us to concentrate on either of the two ideas in order to study complementation. Recalling, that we stated previously that the difference between fuzzy sets and crisp sets lies in the former's lack of satisfying the law of complementation. Since we have just seen that complementation is in essence equivalent to a complete distinction between $A$ and $\bar{A}$ we can now isolate the aspect which distinguishes fuzzy subsets from ordinary subsets. In particular, we note that the requirement of distinction between $A$ and $\bar{A}$ is what is not satisfied by fuzzy sets. That is, the essence of fuzziness lies in the lack of distinction between $A$ and its negation $\bar{A}$. Therefore, we suggest that any measure of fuzziness should be a measure of the lack of distinction between $A$ and $\bar{A}$. More generally, if $A$ is some concept or idea then this concept is fuzzy when it doesn't distinguish clearly between $A$ and not $A$.†

Thus we propose that the search for measures of fuzziness should be guided by a search for

†As we have shown there is a mathematical duality between exhaustiveness and distinction in situations where negation satisfies involution and De Morgan's laws. However, from an intuitive point of view it appears more natural to discuss fuzziness in terms of the lack of distinction between things rather than a lack of exhaustiveness. In many instances one can see apparent contradictions without having all the information necessary to say that these things are not exhaustive.

measures of distinction between $A$ and not $A$. That is, the more distinct $A$ and $\bar{A}$ the less fuzzy $A$.

## MEASURES OF FUZZINESS

In this first part we confine ourselves to fuzzy subsets of the type initially defined by L. A. Zadeh.[11] That is, we consider fuzzy subsets whose grades of membership lie in the unit interval.† Following Zadeh,[11] if $A$ is a fuzzy subset of $X$ then its negation, $\bar{A}$, is also a fuzzy subset of $X$ in which

$$\bar{A}(x) = 1 - A(x) \quad \text{for all} \quad x \in X.$$

As discussed in the previous section a measure of fuzziness of $A$ should be a measure of the lack of distinction between $A$ and $\bar{A}$. The closer $A$ and $\bar{A}$ the more fuzzy $A$.‡ In this section we investigate various metric methods for measuring the fuzziness of $A$ and hence the lack of distinction between $A$ and $\bar{A}$. In particular, if $A$ is a fuzzy subset of $X$ we shall allow $f(A)$ to be its measure of fuzziness and therefore $f(A)$ is some function of the lack of distinction between $A$ and $\bar{A}$. We note that we require the function $f$ to satisfy $f(A) = f(\bar{A})$.

Having asserted that a measure of fuzziness of a fuzzy subset $A$ is the lack of distinction between $A$ and $\bar{A}$ the first question that arises is how to measure this lack of distinction. The first method which comes to mind is to measure the distance between $A$ and $\bar{A}$. Therefore, let us consider some metric type measures of fuzziness based on the distance between $A$ and $\bar{A}$. Consider first the case where the underlying set $X$ is finite of size $n$. In this case we can consider each fuzzy subset of $X$ as a vector with $n$-components. That is, $A(x_i)$ is the $i$th component of a vector representing the fuzzy subset $A$. In this case then $(1 - A(x_i))$ is the $i$th component of the vector representing the fuzzy subset $\bar{A}$. Using this structure we can consider lack of distinction to be measureable by the distance between $A$ and $\bar{A}$. That is, the closer $A$ and $\bar{A}$ the less distinction and hence the greater the fuzziness of $A$. Therefore, using these ideas we can choose any metric on the space of $n$ vectors to help measure the lack of distinction necessary to formulate $f(A)$. Thus, if $D$ is a metric on the $n$ space then $D(A, \bar{A})$ is a measure of distance between $A$ and $\bar{A}$. We note that $f(A)$ is not equal to $D(A, \bar{A})$. A class of metrics in this space is defined by

$$D_p(A, A) = \left[ \sum_{i=1}^{n} |AC(x_i) - A(x_i)|^p \right]^{1/p}$$

for $p = 1, 2, 3\ldots$ .

In particular, we have,

$p = 1$: Hamming metric, where

$$D_1[A, \bar{A}] = \sum_{i=1}^{n} |A(x_i) - \bar{A}(x_i)|.$$

However, since $\bar{A}(x) = 1 - A(x)$, we get

$$D_1[A, \bar{A}] = \sum_{i=1}^{n} |2A(x_i) - 1|.$$

$p = 2$: Euclidean Metric, where

$$D_2[A, \bar{A}] = \left( \sum_{i=1}^{n} (A(x_i) - \bar{A}(x_i))^2 \right)^{\frac{1}{2}}$$

and since $\bar{A}(x) = 1 - A(x)$ we get

$$D_2[A, \bar{A}] = \left( \sum_{i=1}^{n} (2A(x_i) - 1)^2 \right)^{\frac{1}{2}}$$

$p = \infty$: Sup Metric

$$D_\infty[A, \bar{A}] = \operatorname*{Sup}_{i=1,2,\ldots,n} |A(x_i) - \bar{A}(x_i)|$$

$$D_\infty[A, \bar{A}] = \sum_{i=1,2,\ldots,n} |2A(x_i) - 1|$$

We note that if $C$ is a crisp set than $\bar{C}$ is truly complementary and the following theorem can be proved.

THEOREM 1. *If $C$ is any crisp subset of $X$ and $\bar{C}$ its negation then*

$$D_p = D_p[C, \bar{C}] = n^{1/p}$$

†In a second part of this paper we shall consider fuzzy subsets with membership grades in lattices.

‡Though we speak of the fuzziness of A, involution implies that $\bar{\bar{A}} = A$ and therefore $\bar{A}$ is equally as fuzzy. However, it appears more natural to speak in terms of $A$.

*in particular*

$$D_1 = n$$
$$D_2 = n^{\frac{1}{2}}$$
$$D_\infty = 1.$$

*Proof* If $C$ is crisp then either $C(x)$ or $\bar{C}(x)$ is 1 and the other is zero and, therefore,

$$|C(x) - \bar{C}(x)| = 1$$

and

$$D_p(C, \bar{C}) = \left( \sum_{i=1}^{n} 1 \right)^{1/p} = n^{1/p}.$$

THEOREM 2. *If $C$ is any crisp subset of $X$ and $A$ is any fuzzy subset of $X$ then*

$$D_p(A, \bar{A}) \leqq D_p(C, \bar{C}).$$

*Proof* $|C(x_i) - \bar{C}(x_i)| = 1$ for all $x_i$ and

$|2A(x_i) - 1| \leqq 1$ for all $x_i$

therefore the theorem follows.

We note that our definition of fuzziness is inversly related to the distance between $A$ and $\bar{A}$. That is, the closer $A$ and $\bar{A}$ the more fuzzy $A$. Based upon these metrics we can define a measure of fuzziness of $A$, $f_p(A)$ such that

$$f_p(A) = \frac{D_p - D_p(A, \bar{A})}{D_p} = 1 - \frac{D_p(A, \bar{A})}{D_p}$$

In particular,

$$f_1(A) = 1 - \frac{1}{n} \left( \sum_{i=1}^{n} |2A(x_i) - 1| \right)$$

$$f_2(A) = 1 - \frac{1}{n^{\frac{1}{2}}} \left( \sum_{i=1}^{n} |2A(x_i) - 1|^2 \right)^{\frac{1}{2}}$$

$$f_\infty = 1 - \operatorname*{Sup}_{i=1,2,3,\ldots} |2A(x_i) - 1|$$

We can see that this definition of fuzziness satisfies the three conditions stipulated by De Luca and Termini:[12]

1) If $A$ is a crisp set, $A(x_1) = 1$ or 0 and therefore $|2A(x_i) - 1| = 1$ for all $i = 1, \ldots n$ and therefore $D_p(A, \bar{A}) = D_p$ and hence

$$f_p(A) = 1 - \frac{D_p}{D_p} = 0.$$

2) $f_p(A) = 1 - (D_p(A, \bar{A}))/(D_p)$ is maximum when $D_p(A, \bar{A})$ is minimum. $D_p(A, \bar{A})$ is minimum when $|2A(x_i) - 1| = 0$ for all $x_i$. Since $|2A(x_i) - 1| = 0$, when $A(x_i) = \frac{1}{2}$, we see that $f_p(A)$ is maximum when $A(x_i) = \frac{1}{2}$ for all $x \in X$.

3) Assume $A^*(x) \geqq A(x)$ for $A(x) \geqq \frac{1}{2}$

$$A^*(x) \leqq A(x) \quad \text{for} \quad A(x) \leqq \tfrac{1}{2};$$

then

$$|2A^*(x_i) - 1| \geqq |2A(x_i) - 1| \text{ for all } x_i$$

and, therefore.

$$D_p(A^*, \bar{A}^*) \geqq D_p(A, \bar{A})$$

and

$$f_p(A^*) \leqq f_p(A).$$

If the underlying set $X$ is continuous, in particular some interval $(a, b)$ of the real line, then we can define fuzziness using an integral measure of distance. That is, If $X = [a, b] \in R$ then if $A$ is a fuzzy subset of $X$, and $\bar{A}$ its negation then

$$D_p(A, \bar{A}) = \left[ \int_a^b |A(x) - \bar{A}(x)|^p dx \right]^{1/p}$$

$$(p = 1, 2, \ldots, \infty).$$

Furthermore since $\bar{A}(x) = 1 - A(x)$ we have

$$D_p(A, \bar{A}) = \left[ \int_a^b |2A(x) - 1|^p dx \right]^{1/p}$$

When $p = 1$ we get

$$D_1(A, \bar{A}) = \int_a^b |2A(x) - 1| \, dx,$$

and when $p = 2$

$$D_2(A, \bar{A}) = \left[ \int_a^b (2A(x) - 1)^2 dx \right]^{\frac{1}{2}}$$

If $C$ is a crisp set of $X$, then

$$|2C(x) - 1| = 1 \quad \text{for all} \quad x$$

and therefore

$$D_p(C, \bar{C}) = \left[ \int_a^b dx \right]^{1/p} = (b - a)^{1/p}.$$

Thus for crisp sets we get

$$D_1 = b - a$$
$$D_2 = (b-a)^{\frac{1}{2}}$$
$$D_p = (b-a)^{1/p}.$$

As in the finite case it can easily be shown that if $A$ is a fuzzy subset of $X$

$$D_p \geqq D_p(A, \bar{A}).$$

Therefore, we have our measure of fuzziness in this case as

$$f_p(A) = 1 - \frac{1}{(b-a)^{1/p}} [\int_a^b |2A(x) - 1|^p dx]^{1/p}.$$

It can easily be shown that this definition of fuzziness measure satisfies the conditions stipulated by De Luca and Termini.[12]

An alternative metric measure of fuzziness is possible based upon the idea of lack of distinction being the measure of fuzziness.

Assume $A$ is a fuzzy subset of $X$ and $\bar{A}$ is its negation. Let the fuzzy subset $F$, be defined as

$$F = A \cap \bar{A},$$

that is, the intersection of $A$ and not $A$. When $A$ is non-fuzzy, $F$ becomes the null set. The further $F$ is from the null set the less distinct $A$ and $\bar{A}$, hence the more fuzzy is $A$. If we use the metric distance between $F$ and $\Phi$ as our measure of fuzziness we obtain the measure of fuzziness suggested by Kaufman.[19] Since $F(x) = \text{Min}[A(x), 1 - A(x)]$, $F(x) \leqq \frac{1}{2}$ for all $x \in X$.

The distance or distinction between $F$ and the null set can be expressed by the usual metric.†

$$D_p(F, \Phi) = \left( \sum_{i=1}^{n} |F(X_i)|^p \right)^{1/p}.$$

When $A$ is crisp, $F(x) = 0$ for all $x$ and $D_p(F, \Phi)$ assumes its minimal value of zero. When $A(x) = \frac{1}{2}$ for all $X$, $F(x) = \frac{1}{2}$ for all $x$ and $D_p(F, \Phi)$ assumes its maximal value $n^{1/p}/2$. Thus we see that

$$0 \leqq D_p(F, \Phi) \leqq n^{1/p} 2.$$

†We shall here consider only the case when $X$ is a finite set. The extension to the case when $X$ is continuous is straightforward.

Recalling that in this case the larger the distinction of $F$ from $\Phi$ the fuzzier $A$ we can normalize $D_p(F, \Phi)$ to obtain a measure of fuzziness of $A$,

$$G_p(A) = 2/n^{1/p} D_p[F, \Phi]$$

Kaufman[19] defines the index of fuzziness of a fuzzy subset as

$$I(B) = 2/n^{1/p} D_p[B, B^c]$$

where $B^c$ is the crisp set nearest to $B$, defined as

$$B^c(x) = 0 \quad \text{if} \quad B(x) \leqq \tfrac{1}{2}$$
$$B^c(x) = 1 \quad \text{if} \quad B(x) > \tfrac{1}{2}$$

Simple substitution will indicate that our measure is the same as Kaufman's.

## NON-METRIC MEASURES OF FUZZINESS

We have only considered metric measures of fuzziness. That is, we have measured the fuzziness of a set by considering measures of distinction based solely upon the distance between two fuzzy subsets. In this section we discuss some measures of fuzziness which are not based upon a metric measure of distinction. In this we use the concept of compatability[11–14] to measure the distinction between fuzzy subsets.[21–23] This approach is also useful in the cases in which the grade of membership structure is that of a lattice. Since this approach leads to linguistic type measures of fuzziness the concept of a linguistic variable[23] is discussed first.

Assume $V$ is a variable which can take values in a set $X$. If we allow, for example, $V$ to be John's height and $X$ to be a set of possible weights John can assume, then we can assign $V = x$, $x \in X$. We shall call $V$ a linguistic variable if we allow $V$ to assume values which are fuzzy subsets of $X$. The concept of linguistic variable comes from the fact that we can express such concepts as tall, very tall, short, etc. as fuzzy subsets of $X$ and thus by allowing $V$ to assume values of fuzzy subsets we are also allowing it to assume these linguistic values.

A particularly significant linguistic variable is the variable truth. That is, if we allow $V$ to be the truth of some statement and allow it to assume values in the set $I$, the unit interval, we can then express truth as a linguistic variable. In essence

truth is a fuzzy subset of the unit interval. The concept of compatibility between two fuzzy subsets was original suggested by Zadeth[21,22] and used extensively by Yager[3,23] in validating complex models. Assume $X$ is a set and $F$ is the class of fuzzy subsets with membership grades taken in the unit interval. If $A \in F$ and $x \in X$ then $A(x) \in I$ is a measure of the grade of membership of $x$ in $A$. Alternatively $A(x)$ can be interpreted as the degree of compatibility of the object $x$ with the concept described by the fuzzy subset $A$. Another interpretation is that $A(x)$ is a measure of the truth of the fact that $x$ satisfies $A$.

If $B$ is also a fuzzy subset of $X$, one can ask what is the degree of membership of $B$ in $A$. That is, how true is it that the set $B$ satisfies the condition stipulated by $A$. From the extension principle[3] we obtain the degree of membership of $B$ in $A$, denoted $A(B)$ as

$$A(B) = \left\{ \begin{matrix} B(x) \\ A(x) \end{matrix} \right\} = T$$

This is called the compatibility of $B$ with $A$.

*Example*

$$X = \{x_1, x_2, x_3, x_4\}$$

$$A = \left\{ \frac{0.7}{x_1}, \frac{0.3}{x_2}, \frac{0.5}{x_3}, \frac{0.5}{x_4} \right\}$$

$$B = \left\{ \frac{0.6}{x_1}, \frac{0.5}{x_2}, \frac{0.4}{x_3}, \frac{0.9}{x_4} \right\}$$

$$A(B) = \left\{ \frac{0.6}{0.7}, \frac{0.5}{0.3}, \frac{0.4}{0.5}, \frac{0.9}{0.5} \right\}$$

$$= \left\{ \begin{matrix} 0.6 & 0.5 & 0.9 \\ 0.7 & 0.3 & 0.5 \end{matrix} \right\} = T$$

We have observed that $T$, $A(B)$, is a fuzzy subset in the unit interval. That is, $T$ can be interpreted as a linguistic truth value taking on such values as "high", "very high", "low", etc. Thus, $T$ would measure the compatability of $B$ with $A$. Zadeh[10] has used this concept of compatibility to develop a procedure for answering fuzzy questions in the face of fuzzy data.

Assume $V$ is some variable which can take on as values fuzzy subsets of $X$. That is, $V$ has values in $F$. Assume we observe empirically that $V = G$. Let us consider the following question "Is $V = H$?", where $H$ is some other fuzzy subset in $X$. As an example if we let $V$ stand for John's height and observe that John is about 6 feet, which is a fuzzy concept, we may ask "Is John very tall?", again a fuzzy concept. Zadeh[10] has suggested that the answer to this question is the truth of the statement $V$ is $H$ in light of the data $V$ is $G$. In particular, he shows that the answer is a linguistic truth variable whose value is equal to the compatibility of $G$ with $H$. That is, the answer to the question "Is John very tall?" is the truth value equal to the compatibility of about six feet with very tall.

We can now use this structure in order to investigate the concept of fuzziness in a fuzzy set. Assume $X$ is a set. Let $V$ be the variable indicating the fuzzy subset we are studying. Assume $A$ is a fuzzy subset of $X$. Then the question of investigating fuzziness becomes, "Given that $V = A$, is $V$ fuzzy?" We must now formulate the question is $V$ fuzzy in a form amenable to the questioning procedure discussed above. In particular, we must obtain a surrogate question for the question "Is $V$ fuzzy?"

Recalling our discussion of fuzziness in the earlier sections, we noted that fuzziness is measured by the lack of distinction between $A$ and $\bar{A}$, which in turn forces $A \cap \bar{A}$ to be distinct from $\Phi$. We can then use this observation to obtain a surrogate question for, "Is $V$ fuzzy?"

In particular we suggest that a measure of fuzziness can be obtained as the answer to the question. "Given $V = A$, is $V = \bar{A}$?" The truer this answer the more fuzzy $A$. Therefore, following the above procedure, we find that the answer is the compatibility of $A$ with $\bar{A}$. That is, $T_1 = \bar{A}(A) = \{A(x)/\bar{A}(x)\}$ for $x \in X$, measures the compatibility of $A$ with $\bar{A}$. It is noted that $T_1$ is a fuzzy number in the unit interval. Assume $A$ and $B$ are two fuzzy subsets over $X$ and we are interested in the question of which is fuzzier $A$ or $B$. We can calculate the compatibility of each with its negation and then use either linguistic approximation[24] or a procedure for ranking fuzzy numbers suggested by Baas and Kwakernaak.[25]

An alternative measure of fuzziness is possible based upon the above ideas. If we let $V = A \cap \bar{A}$ and ask "Is $V = I$?" In this case we get the compatibility of $A \cap \bar{A}$ with $I$, that is

$$T_2 = \left\{\frac{A \cap \bar{A}}{I}\right\} = \left\{\frac{\min[A(x), 1 - A(x)]}{I(x)}\right\}$$

for all $x \in X$;

however since $I(x) = 1$ for all $x$, then

$$T_2 = \operatorname{Sup}_{x \in X} \operatorname{Min}[A(x), 1 - A(x)]$$

In this case, $T_2$ is a number. Furthermore, since the larger $T_2$ the fuzzier $A$ we see that a simple method of comparing fuzzy sets with respect to fuzziness is available. That is, if $A_1, A_2, \ldots A_n$ are a set of fuzzy subsets of $X$, they can be ordered with respect to their fuzziness by $T_2$. Another non-metric measure of fuzziness is possible based upon these ideas. If we let $F = A \cap \bar{A}$ and $G = A \cup \bar{A}$, then the more compatible these two sets the more fuzzy $A$, that is

$$T_3 = \left\{\frac{G(x)}{F(x)}\right\} \quad \text{for all} \quad x \in X.$$

Again we note that $T_3$ gives us a linguistic measure of fuzziness. That is, $T_3$ is a linguistic truth value answer to the question "Is $A$ fuzzy?"

Thus we have developed three non-metric measures of fuzziness.

## CONCLUSION

We have introduced what we feel is a holistic idea of the measurement of fuzziness. This idea being that fuzziness is related to the lack of distinction between a concept and its negation. We have used this idea to develop various measures of fuzziness of fuzzy subsets with grades of membership in the unit interval. In a subsequent paper we shall apply this idea of measuring fuzziness to subsets with grades of membership in lattices.

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# PROCEDURES FOR GENERATING RECONSTRUCTION HYPOTHESES IN THE RECONSTRUCTABILITY ANALYSIS†

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The reconstructability analysis is viewed in this paper as a set of procedures whose objective is to identify from among all possible collections of subsystems of a given overall system those which allow the reconstruction of specific properties of the overall system under given quality criteria. The paper focuses on algorithmic details of procedures for the generation of meaningful reconstruction hypotheses. It also describes the role of these generating procedures in the overall reconstructability analysis, as well as the place of the reconstructability analysis in the larger context of computer-aided systems problem solving. Included in the paper are FORTRAN IV definitions of the generating procedures, and a reference is made to similar definitions in the APL computer language.

INDEX TERMS: Reconstructability analysis, general systems problem solver, structure system, structure refining procedure, structure aggregating procedure, reconstruction hypothesis.

## 1 INTRODUCTION

The human mind, when encountering very complex systems, seems to have certain faculties which make it superior even to the most sophisticated formal methods applied on the most advanced computers. The current understanding of these faculties is rather rudimentary, and certainly unsatisfactory, in spite of the tremendous efforts, primarily in the area of artificial intelligence, to model them and develop their machine counterparts.

Intuition, insight and the ability of global comprehension are possibly the most valuable assets of the human mind, particularly one that is appropriately trained. However, complex systems frequently possess properties which are counterintuitive and resistant to global comprehension. As such, they represent traps for the human mind in the sense that they may guide the human mind into illusory insights. To discover such traps, it is often unavoidable to perform the tedious work of detailed analysis of the system on hand. While the human mind is weak and severely limited in this respect, it is exactly this domain of detailed analysis where the computer, equipped with appropriate formal methods, excels. This ability gives the computer an important role as an *intuition safeguard* and *intuition amplifier*.[5]

In contrast with the area of artificial intelligence, where the emphasis has traditionally been on replacing the human mind by a machine, the basic philosophy of systems methodology is to supplement the human mind, in a symbiotic fashion, by the computer equipped with an organized package of appropriate methodological tools. Such philosophy is more explicitly exposed in previous works by Cavallo, Klir and Uyttenhove[2,3,11] which describe an overall conceptual framework for computer-aided systems problem solving as well as basic characteristics of an associated man-machine complex, referred to as **General Systems Problem Solver** or **GSPS**.

The symbiosis of the human being (investigator, decision maker and the like) with the methodologically equipped computer, such as the

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GSPS, makes it possible to invent and utilize new approaches to various intellectual tasks, far superior to those which are applicable by either of them alone. The strength of the human being is his experience in the area of study, understanding and taking advantage of the context of investigation, intuition, global comprehension, feeling for the right solution, creativity, and the like. The strength of the computer lies in its computational power, the ease with which it can handle tremendous numbers of operations, far exceeding the human capability in this respect. It is the computational power which, when properly utilized, can significantly enhance the human intellectual qualities by providing the human being with desirable detailed analyses and, as already mentioned, help him to avoid the many counterintuitive traps associated with complex systems.

One such counterintuitive trap is seeded in the assumption, often taken for granted, that properties of overall systems can be reconstructed from knowledge of corresponding properties associated with their subsystems. Such assumption is unfortunately warranted only in the case of some systems and, even if it is warranted, its validity depends on the subsystems involved.

If the assumption of the ability to reconstruct an overall system from its specific subsystems is not warranted, the various conclusions about the overall system obtained from the subsystems may be vastly incorrect and misleading. Although information about the reconstruction possibilities is implicitly included in data regarding the overall system, their discovery requires a detailed analysis of the data. While such analysis can well be carried out by the computer, it is next to impossible for the human mind. Let this analysis be referred to as **reconstructability analysis**.

The objective of the reconstructability analysis, as introduced in a previous paper,[4] is to identify from among all possible collections of subsystems of a given overall system those which allow the reconstruction of specific properties of the overall system under given quality criteria. The reconstructability analysis consists of procedures for generating meaningful reconstruction hypotheses, procedures for the evaluation of the reconstruction hypotheses, and procedures for making various decisions regarding the acceptance of evaluated reconstruction hypotheses, generation of additional reconstruction hypotheses, termination of the analysis and the like.

This paper focuses on the procedures for generating structure systems which can be used as meaningful reconstruction hypotheses in the reconstructability analysis. It is fully based on concepts and algorithmic schemes described in the mentioned paper by Cavallo and Klir.[4] Its primary objective is to elaborate upon the algorithmic schemes and define their specific implementations in a standard programming language. From two currently available implementations, one in FORTRAN IV and one in APL, we decided to use the one in FORTRAN IV in this paper and make the APL implementation available through the General Systems Depository.[6]

From a larger perspective, the procedures described in the paper are only one part of a package of procedures for the reconstructability analysis, and this package is in turn a part of a still larger package of methodological tools for systems problem solving. The latter package is currently being developed within the previously mentioned conceptual framework of GSPS. As a contribution to the GSPS development, this paper also describes the overall GSPS scheme and indicates the role of the reconstructability analysis in this larger scheme. Emphasis is on the development of GSPS software rather than on various theoretical or application oriented questions.

## 2 RELEVANT CONCEPTS AND PREVIOUS RESULTS

The **overall system** which is a subject of the reconstructability analysis is defined as the sixtuple[4]

$$B = (V, \mathscr{V}, s, A, Q, b), \tag{1}$$

where $V = \{v_i | i \in N_n\}$ $(N_n = \{1, 2, \ldots, n\})$ is a set of *variables*; $\mathscr{V} = \{V_j | j \in N_m, m \leq n\}$ is a family of *state sets*; $s: V \to \mathscr{V}$ is an onto *assignment function* by which one state set from $\mathscr{V}$ is assigned to each variable in $V$; $A = s(v_1) \times s(v_2) \times \ldots \times s(v_n)$ is the set of all *potential aggregate states; Q* is a set of real numbers; $b: A \to Q$ is a function which represents a characterization of the *actual aggregate states* of the overall system, typically a selection function, probability distribution function, or fuzzy set membership function.

A *subsystem* of an overall system is defined in the same general form (1) but is based on a subset of variables of the overall system and the projection of function $b$ which disregards all variables in set $V$ except those which are in the subset. A set of subsystems is called a *structure system.*

In principle, structure systems are viewed as reconstruction hypotheses. The task of the reconstructability analysis is to generate them and evaluate their reconstruction abilities with respect to the objectives of the reconstructability problem. It follows from the definition of the structure systems[4] that, given an overall system and a specific formulation of the reconstructability problem, each family of subsets of variables of the overall system uniquely identifies a structure system. When only procedures for generating structure systems as reconstruction hypotheses are the subject of interest, like in this paper, such identification is sufficient and advantageous. In order to distinguish this identification of structure systems from the actual structure systems, families of subsets of variables are referred to as *structures*, in conformity with the preceding paper.[4]

Let $V$ be a set of variables which represents a system under investigation. Then, for the purpose of this paper, a general structure acceptable as a **reconstruction hypothesis**, say $G_i$, is viewed as a family of subsets of $V$ which satisfies the following properties

i) $\bigcup_{E_a \in G_i} E_a = V$;

ii) $E_a \not\subset E_b$ for each pair $E_a, E_b \in G_i (a \neq b)$.

Property (i) guarantees that all variables of the overall system are involved in the reconstruction hypothesis. Property (ii) guarantees that no subsystem is redundant in the sense that it contains only variables which are included in any other subsystem of the same structure; such redundant subsystem would contribute no information about the overall system which is not present in some other subsystem of $G_i$.

Observe that structures which satisfy properties (i) and (ii) are *hypergraphs.*[1]

Let $S_G$ denote the set of all *general structures* or *G-structures* (structures which satisfy properties (i) and (ii)). Let $S_G$ be mapped onto the set $R$ of all reflexive and symmetric binary relations (undirected graphs with loops, compatibility or tolerance relations) defined on set $V$ by a function

$$r: S_G \rightarrow R$$

such that $r(G_i)$ is the binary relation in which variables $v_j$ and $v_k$ are related if and only if they both belong to at least one of the subsets of $V$ through which $G_i$ is defined. We shall refer to the binary relations in $R$ as *graphs* and to the structures in $S_G$ as *hypergraphs.*

Function $r$ imposes the following equivalence relation on the set $S_G$ of all general structures:

$$G_i \equiv G_j \quad \text{iff} \quad r(G_i) = r(G_j)$$

Let $S_G/r$ denote the set of all equivalence classes imposed on $S_G$ by $r$.

It is imperative for the reconstructability analysis that reconstruction hypotheses be ordered by a *refinement relation.* For each pair $G_i, G_j \in S_G$, let $G_i \leqq G_j$ if and only if for each set $E_a \in G_i$ there exists a set $E_b \in G_j$ such that $E_a \subseteq E_b$. When $G_i \leqq G_j$, $G_i$ is called a *refinement* (or *successor*) of $G_j$ or, alternatively, $G_j$ is called an *aggregate* (or *predecessor*) of $G_i$. Moreover, $G_i$ is called an *immediate refinement* (or *immediate successor*) if and only if $G_i \leqq G_j$ and there is no $G_k$ such that $G_i \leqq G_k$ and $G_k \leqq G_j$; the term "*immediate aggregate*" (or "*immediate predecessor*") is defined similarly.

It is well known[4] that $(S_G, \leqq)$ is a lattice. Similarly, set $R$ together with the relation of "being a subset of" form a lattice, and the one-to-one correspondence between $R$ and $S_G/r$ imposes an isomorphic lattice on set $S_G/r$. This isomorphism makes it possible to generate equivalence classes in $S_G/r$ by appropriately operating on graphs in $R$. It is desirable, primarily for computational reasons, that each equivalence class in $S_G/r$ be uniquely represented not only by a graph but also by a canonical structure (hypergraph). There are two subsets of $S_G$ which qualify for this purpose:[4]

1) set $S_C$ which consists of such general structures $G_i$ that contain only and all maximal compatibility classes of $r(G_i)$;[10]

2) set $S_P$ which consists of such general structures $G_i$ that contain no compatibility classes of $r(G_i)$ except all those which consist of two variables and all maximal compatibility classes with single variables (isolated vertices in the graph).

Let structures in $S_C$ and $S_P$ be called *C-structures* and *P-structures*, respectively. It can easily be shown[4] that each equivalence class in $S_G/r$ contains exactly one *C*-structure and one *P*-structure. Furthermore, the *C*-structure is the least refined structure in the equivalence class and the *P*-structure is the most refined one. Two functions (one-to-one)

$$r_C : S_C \rightarrow R$$

$$r_P : S_P \rightarrow R$$

can be defined in exactly the same way as function $r$ but for the restricted domain $S_C$ or $S_P$, respectively.

Some additional useful subsets of $S_G$ were introduced in the preceding paper.[4] One of them consists of all such *G*-structures $G_i$ which contain only maximal compatibility classes of $r(G_i)$, even though not necessarily all of them. They are called *M-structures*[4] and the set of all of them is denoted by $S_M$. Another subset of *G*-structures, denoted by $S_I$, consists of all such *G*-structures $G_i$ which represent irredundant covers of $r(G_i)$; these structures are referred to as *I-structures*.[4] The significance of the *M*-structure and *I*-structures for the generation of reconstruction hypotheses is discussed in Section 4.10.

Two remaining subsets of $S_G$ introduced in the mentioned paper,[4] which are referred to as $\beta$-structures and $\gamma$-structures, were motivated by various investigative contexts and considerations regarding the evaluation of reconstruction hypotheses. They have no significance for developing procedures for generating reconstruction hypotheses and, therefore, they are not discussed in this paper.

## 3 OVERALL SCHEME

As mentioned in Section 1, GSPS[2,3,11] is based on a symbiosis between a human being, referred to as *user*, and a methodologically equipped computer, viewed as *GSPS proper* (Figure 1).

The GSPS proper consists of packages of methodological tools for systems problem solving which are expressed in terms of computer software and organized according to a conceptual framework through which admissible classes of systems, requirements (objectives, constraints) and systems problems are defined. The conceptual framework is also a basis for the interaction between the user and GSPS proper. The interaction involves a set of procedures, referred to as identification procedures (Figure 1), through which the user is assisted in formulating his problem.

As shown in Figure 1, one of the packages of methodological tools is referred to as "*core (general) package*", while the others are called "*specialized packages*". The core package is reserved for the most general methods, which have the broadest applicability within the conceptual framework; the specialized packages consist of variations of the general methods, each based on a set of particular assumptions which restrict the applicability of the method while, at the same time, make it computationally more efficient than the corresponding general method.

The software implementation of the procedures for the reconstructability analysis, as formulated in this paper, is a part of the core package.

The development of the conceptual framework and the core methodological package, as well as the integration of existing specialized packages into the GSPS framework, have been the primary interests of a team at the Department of Systems Science, SUNY at Binghamton, with which the authors of this paper are affiliated, during the last decade or so.[2,3,7,9,11]

The core package consists of methodological tools for problems associated with systems with nominal scale variables, well defined or fuzzy. The problems are organized in terms of a hierarchy of epistemological systems levels.[3,9,11] They consist of problems which involve a transition from one level to another and problems associated with a single level. A summary of problems for which software is currently available is given in Figure 2; a previous paper is needed for understanding the terminology used.[3]

The reconstructability analysis involves four problems, identified in Figure 2 by ⑤—a transition from an overall (generative or behavioral) system into a structure system which is acceptable as a reconstruction hypothesis, by ⑥—the inverse transition in which the structure system is evaluated, by ④—a comparison of the overall generative system represented by the evaluated structure system with the original (given) overall generative system, and by ⑦—a transition from one structure to its immediate refinements or aggregates.

A general procedural scheme of the recon-

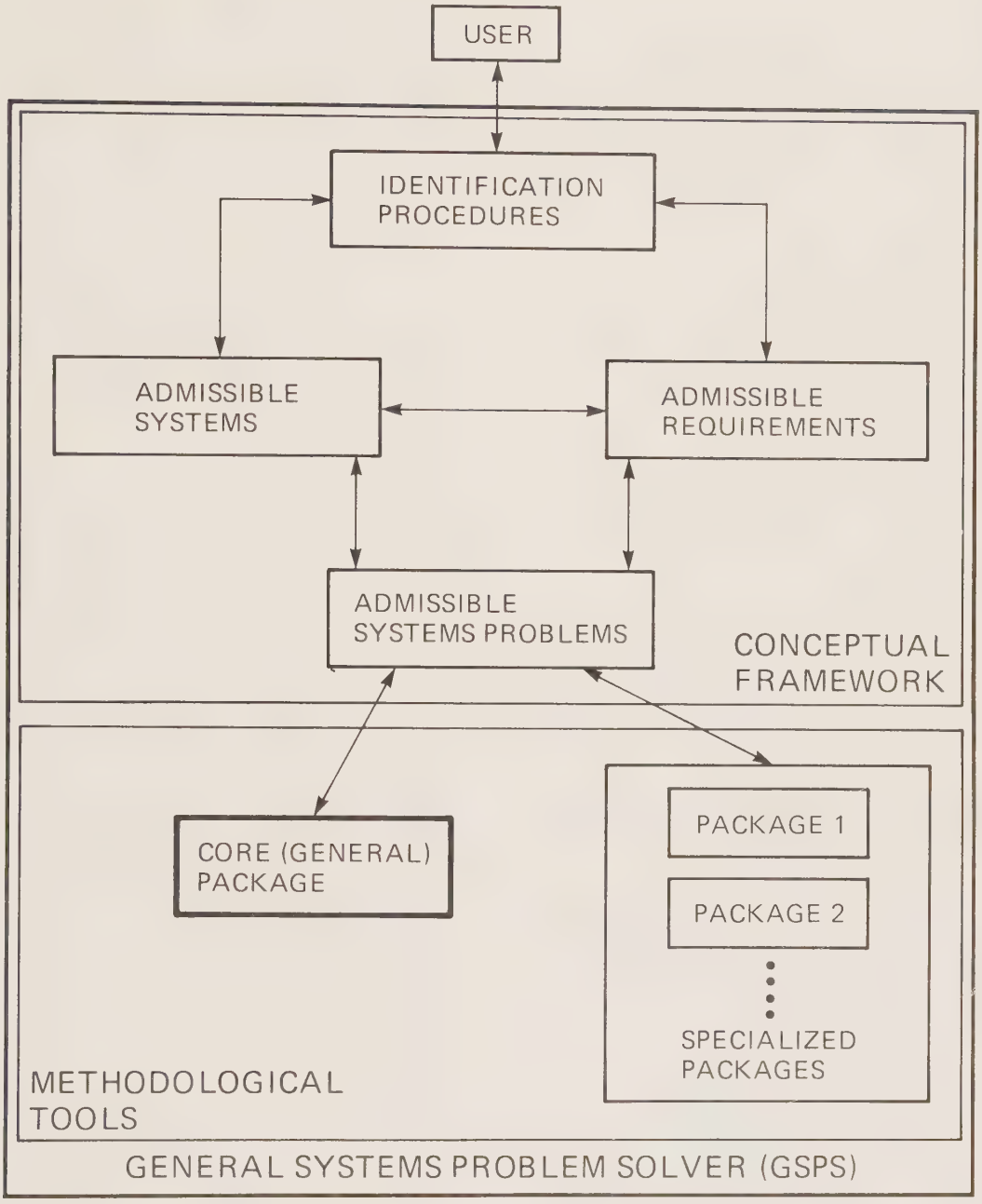


FIGURE 1 General scheme of the General Systems Problem Solver (GSPS).

structability analysis is shown in Figure 3. The first block in the scheme is of primary interest in this paper; it is described in all details in Section 4. The second and third blocks will be described in two subsequent papers.

## 4 PROCEDURES FOR GENERATING RECONSTRUCTION HYPOTHESES

It is clearly desirable to generate reconstruction hypotheses in an orderly fashion, either in the

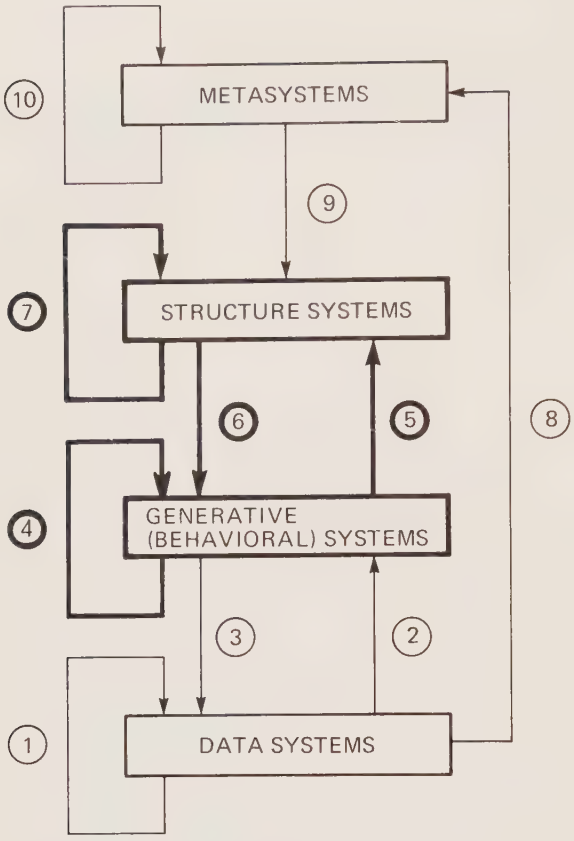


FIGURE 2 Core (general) package of methodological tools (fundamental problem types are identified by the encircled numbers).

increasing order or in the decreasing order of refinement. This can be accomplished by a pair of procedures which for any given general reconstruction hypothesis generate all its immediate refinements and all its immediate aggregates, respectively.

To achieve high computational efficiency and application flexibility, two refining and two aggregating procedures were suggested in the preceding paper.[4] One of them is global† in the sense that it refines or aggregates equivalence classes of structures, as defined by the function $r_G$; the other one is local† in the sense that it refines or aggregates structures within a particular equivalence class.

The *global procedures* (also called *basic procedures*) are based either on the *C*-structures or *P*-structures as canonical representatives of the equivalence classes. Hence, there are four global (basic) procedures: refining procedure for *C*-structure or *RC-procedure*, aggregating procedure for *C*-structure or *AC-procedure*, refining procedure for *P*-structures or *RP-procedure*, and aggregating procedure for *P*-structures or *AP-procedure*.

It is often necessary to refine or aggregate two or more structures at the same level of refinement. In such cases, a procedure is needed which guarantees that no duplicate structures are generated. Moreover, procedures for testing whether the generated structures belong to the various special classes of structures[4] should be included for the benefit of the user.

The *local procedures* (refining or aggregating), which are referred to in the previous paper[4] as *supplementary procedures*, generate for any given general structure all its immediate refinements (or, alternatively, all its immediate aggregates) which are in the same equivalence class as the given structure. Hence, there are two supplementary (local) procedures: refining supplementary procedure or *RS-procedure*, and aggregating supplementary procedure or *AS-procedure*. If desirable, either of these procedures can easily be extended into a procedure which generates all immediate refinements or aggregates, respectively, regardless of the equivalence class. These latter procedures are referred to as refining procedure for general structures or *RG-procedure*, and aggregating procedure for general structures or *AG-procedure*. All of these procedures have to be coupled with a procedure which prevents a generation of duplicates. Moreover, procedures for testing special properties[4] of the generated structures are also desirable.

In some cases, particularly when the number of variables is large, it is desirable to use additional procedures based on *M*-structures. These procedures, which are described in Section 4.10, may be viewed as "*semi-global*" in the sense that they involve a global search in the individual equivalence classes in $S_G/r$.

All the mentioned procedures are schematically summarized in Figure 4. The packaging into global, semi-global and local procedures is also shown in this figure. In an *interactive mode* of the reconstructability analysis, the procedures can be used in any order and with any initial reconstruction hypotheses in mind. In an *automatic*

†The terms "global" and "local", which quite well characterize the different roles of the individual procedures, were suggested by Gerrit Broekstra in a private letter.

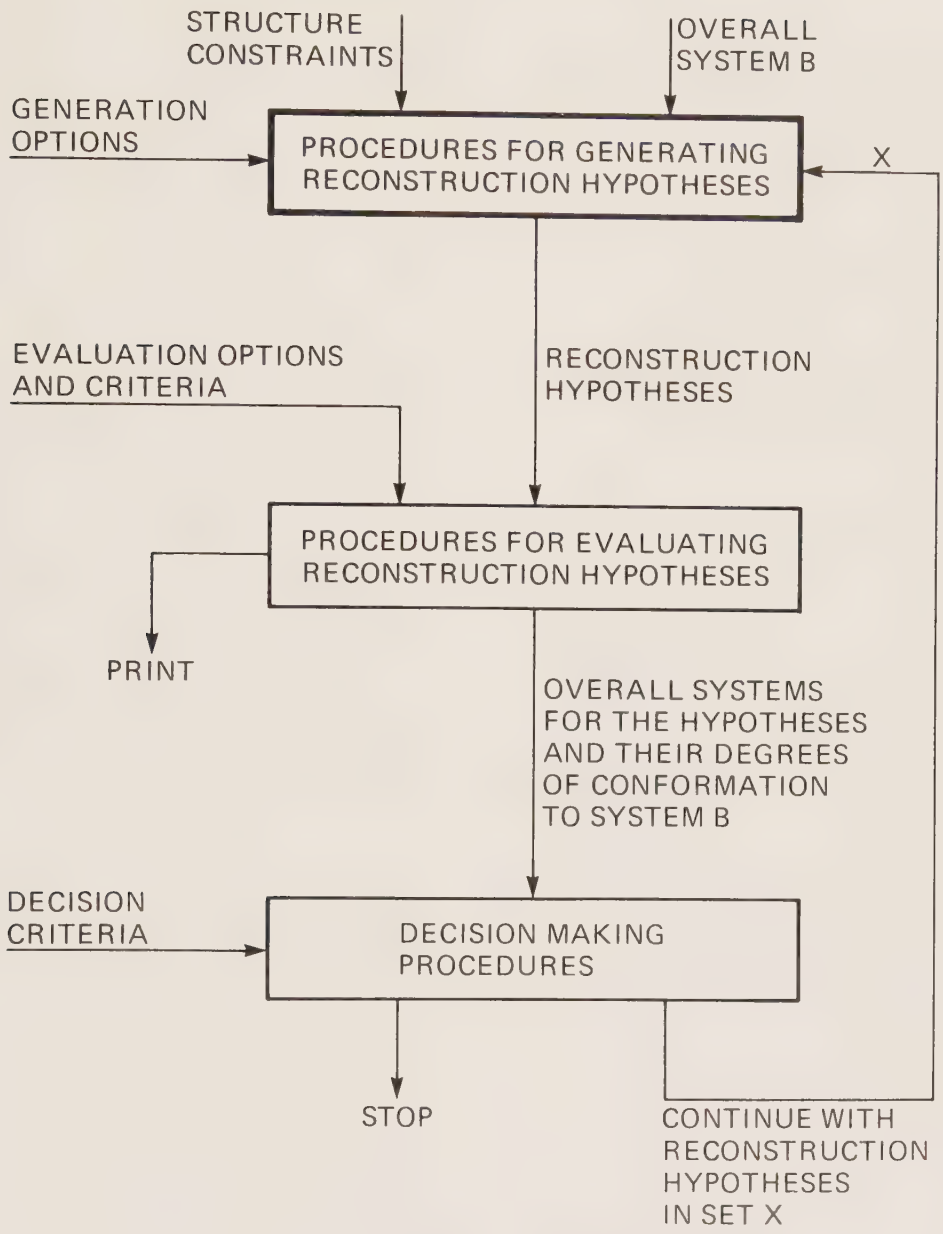


FIGURE 3 General procedural scheme of the reconstructability analysis.

(*batch*) *mode* of analysis, the global procedures are applied first, and are followed either directly by the local procedures or by a sequence of a semi-global procedure and a local procedure. If the user does not indicate any preference, the former alternative is chosen by default when the number of variables is smaller than some fixed number, say $m$, and the latter procedure is used when the number of variables is greater than or equal to that number. Since the application of the $M$-procedures is meaningful only for systems with six variables or more (Section 4.10), it is desirable to require that $m \geqq 6$.

The interactive mode of the reconstructability

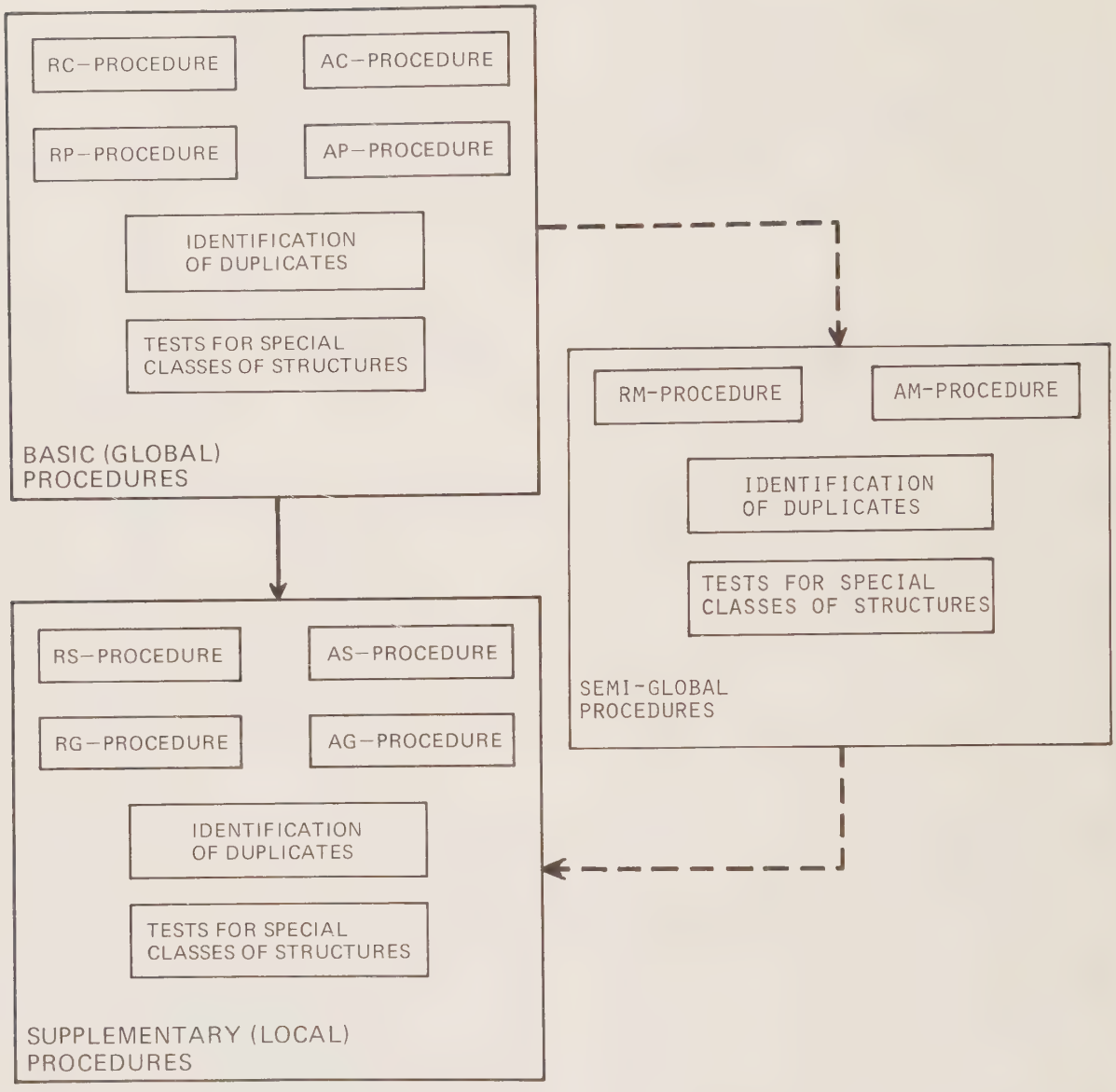


FIGURE 4 Procedures associated with the generation of reconstruction hypotheses.

analysis is basically represented by the mentioned APL software package;[6] the batch mode is represented by the FORTRAN IV package.

In each of the following subsections, one of the procedures is presented both in the form of a rough algorithmic scheme, adopted from the preceding paper[4] and in the form of a detailed FORTRAN IV definition.

The individual listings of the subroutines in FORTRAN IV (Tables I–V) do not include some non-executable statements in order to keep the presentation concise. The following are variable names which are common to all the programs:

$NV$ = The number of variables involved (same as $n$ used in the description of the overall system (1)).

$G$ = A graph representation of structures in a matrix form whose dimension is ($MAXG$, 17), where $MAXG = n(n-1)/2$. The first two columns in the matrix list all unordered pairs of the variables involved

(all potential edges in the graphs). Each of the remaining columns in the matrix represents a selection function through which a particular graph is defined; entries in these columns are 0's or 1's.

$H$ = A hypergraph representation of a structure in a matrix form. Its maximum dimension is $(MAXG, NV)$; its actual dimension is $(MAXH, NV)$, where $MAXH$ is the actual number of subsets of variables in the hypergraph. Each column in $H$ represents one variable and each row represents a selection function through which a particular subset of variables is defined (entries are 0's and 1's). Variable names $KH$ and $SMAXH$ are duplicates of $H$ and $MAXH$, respectively, which are needed for storing the previous contents of $H$ and $MAXH$.

$NG$ = An identifier of a particular graph in matrix $G$.

$TYPE$ = The type of structure which is generated.

All calls to subroutines which handle decisions, printing, storing and evaluation are not included in the program listings in Tables I–V.

### 4.1. *RC-Procedure*

**Algorithmic scheme of the *RC*-procedure.** *Given* $C_k \in S_C$ *and* $\rho_k = r_C(C_k)$, *to generate all immediate refinements of* $C_k$ *in class* $S_C$: (1) exclude one edge (arc) from graph $\rho_k$, say edge $(v_i, v_j)$, where $v_i \neq v_j$ and $(v_i, v_j) \in \rho_k$; (2) split each $E_a \in X$, where $X = \{E_a | E_a \in C_k,\ E_a \supseteq \{v_i, v_j\}\}$, into two subsets $E'_a = E_a - \{v_i\}$ and $E''_a = E_a - \{v_j\}$, and let $Y$ denote the family of sets $E'_a$ and $E''_a$ for all $E_a \in X$; (3) determine $Y' = Y - \{E_\alpha | E_\alpha \in Y,\ \exists (E_b \in C_k - X)(E_\alpha \subseteq E_b)\}$ and record $(C_k - X) \cup Y'$ as an immediate refinement of $C_k$; (4) repeat steps (1)–(3) for all edges in $\rho_k$.

*Program step specifications* (Table I):

*Step* (1): Executed in loop 1000 on the condition that $G(IG, NG+2) \neq 0$.

*Step* (2): Executed in loop 100 on the condition that $N = 2$ in loop 25. Each element to be split is identified as $H(L, I)$, $I = 1, 2, \ldots, NV$. $MAXH$ is increased and identifies a new element $H(MAXH, I)$ composed of $H(L, I)$ without $G(IG, 1)$; $H(L, I)$ is retained without $G(IG, 2)$.

*Step* (3): Subsets are eliminated among those elements referenced in LIST. Subroutine SUBST2 checks the conditions.

*Step* (4): Variable name $IG$ identifies the next edge in $G$ to be excluded.

TABLE I FORTRAN IV program for the *RC*-procedure.

```
      SUBROUTINE RCPROC(IF)
      DO 5 IG=1,MAXG
    5 T(IG)=1
      IF(NG.EQ.1) GO TO 9
      T(1)=0
      CALL DUPLIC(NG,T)
    9 CONTINUE
      DO 1000 IG=1,MAXG
      IF((G(IG,NG+2)*T(IG)).EQ.0) GO TO 1000
C
C     EXCLUDE ONE RELATION IN LOOP 100
C
      SMAXH=MAXH
      DO 10 I=1,MAXH
      DO 10 J=1,NV
   10 KH(I,J)=H(I,J)
      IL=0
      DO 100 L=1,SMAXH
      N=0
      DO 25 I=1,NV
      IF(G(IG,1).NE.(H(L,I)*I)) GO TO 20
      N=N+1
      GO TO 25
   20 IF(G(IG,2).NE.H(L,I)*I) GO TO 25
      N=N+1
   25 CONTINUE
      IF(N.NE.2) GO TO 100
   26 CONTINUE
      IL=IL+2
      LIST(IL)=MAXH+1
      LIST(IL-1)=L
      MAXH=MAXH+1
      DO 40 I=1,NV
      IF(I.EQ.G(IG,1))GO TO 30
      H(MAXH,I)=H(L,I)
      GO TO 40
   30 H(MAXH,I)=0
   40 CONTINUE
      H(L,G(IG,2))=0
  100 CONTINUE
      IF(SMAXH.EQ.1)GO TO 111
C
C      ELIMINATION OF SUBSETS
C
      LH=IL/2
      DO 110 I=1,LH
      IE(1)=LIST((I*2)-1)
      IE(2)=LIST(I*2)
      CALL SUBST2(NV,IE,MV,LMV,0)
  110 CONTINUE
      MAXH=SMAXH
      DO 910 I=1,MAXH
      DO 910 J=1,NV
  910 H(I,J)=KH(I,J)
 1000 CONTINUE
      RETURN
      END
```

### 4.2 *RP-Procedure*

**Algorithmic scheme of the *RP*-procedure.** *Given* $P_k \in S_P$ *and* $\rho_k = r_P(P_k)$, *to generate all immediate*

*refinements of $P_k$ in class $S_P$:* (1) exclude one edge from graph $\rho_k$, say edge $(v_i, v_j)$, where $v_i \neq v_j$; (2) construct $P'_k = P_k - \{\{v_i, v_j\}\}$; (3) let $X = \cup E_a$ for all $E_a \in P'_k$; if $v_i \notin X$ or $v_j \notin X$, include $\{v_i\}$ or $\{v_j\}$ in $P'_k$, respectively, and record $P'_k$ as an immediate refinement of $P_k$; (4) repeat steps (1)–(3) for all edges in $\rho_k$.

The FORTRAN subprogram for the *RP*-procedure is a subset of the subprogram for the *RC*-procedure (Table I). States from 26 to 110 are replaced by

```
      DO 125 K=1,NV
125   H(L,K)=0
```

### 4.3 *AC-Procedure*

**Algorithmic scheme of the *AC*-procedure.** *Given $C_k \in S_C$ and $\rho_k = r_C(C_k)$, to generate all immediate aggregates of $C_k$ in class $S_C$:* (1) include one edge into graph $\rho_k$, say edge $(v_i, v_j)$, where $v_i \neq v_j$ and $(v_i, v_j) \notin \rho_k$; (2) for each $E_x \in C_k$ such that $v_i \in E_x$ and for each $E_y \in C_k$ such that $v_j \in E_y$ construct the set $\{v_i, v_j\} \cup (E_x \cap E_y)$; let $Z$ denote the family of all these sets, and let $C'_k = C_k \cup Z$; (3) for each $E_a \in C_k$ and each $E_b \in Z$, if $E_a \subset E_b$, then exclude $E_a$ from $C'_k$, and for each distinct ordered pair $E_\alpha$, $E_\beta \in Z$, if $E_\alpha \subset E_\beta$, then exclude $E_\alpha$ from $C_k$; record the result of this step as an immediate aggregate of $C_k$; (4) repeat steps (1)–(3) for all edges which are not included in $\rho_k$.

*Program step specifications* (Table II):

*Step* (1): Executed in loop 1000 on the condition that $G(IG, NG+2) \neq 1$.

*Step* (2): Represented by loop 100, index I1, which finds a particular $E_x$ (or $E_y$) and loop 75, index 12, which finds then all the corresponding $E_y$'s (or $E_x$'s, respectively). Loop 40 within loop 75 creates the new compatibles (elements of set $Z$).

*Step* (3): Executed in loop 150.

*Step* (4): Variable name *IG* identifies the next edge in $G$ to be added.

### 4.4 *AP-Procedure*

**Algorithmic scheme of the *AP*-procedure.** *Given $P_k \in S_P$ and $\rho_k = r_P(P_k)$, to generate all immediate aggregates of $P_k$ in class $S_P$:* (1) include one edge into graph $\rho_k$, say edge $(v_i, v_j)$, where $v_i \neq v_j$ and $(v_i, v_j) \notin \rho_k$; (2) construct $P'_k = P_k \cup \{\{v_i, v_j\}\}$; (3) if $\{v_i\} \in P'_k$ or $\{v_j\} \in P'_k$, exclude $\{v_i\}$ or $\{v_j\}$ from $P'_k$, respectively, and record the result of this step as an immediate aggregate of $P_k$; (4) repeat steps (1)–(3) for all edges which are not included in $\rho_k$.

The FORTRAN subprogram for the *AP*-procedure is a subset of the subprogram for the *AC*-procedure (Table II). Statements 11 through 40 and statements 75 through 150 are not used.

TABLE II FORTRAN IV program for the *AC*-procedure.

```
      SUBROUTINE ACPROC(IP)
      DO 5 IG=1,MAXG
5     T(IG)=1
      IF(NG.EQ.1) GO TO 9
      T(1)=1
      CALL DUPLIC(NG,T)
9     CONTINUE
      DO 1000 IG=1,MAXG
      IF((G(IG,NG+2)*T(IG)).EQ.1) GO TO 1000
      SMAXH=MAXH
      DO 10 I=1,MAXH
      DO 10 J=1,NV
10    KH(I,J)=H(I,J)
11    CONTINUE
      MAX=MAXH-1
      KMAX=MAXH
      DO 100 I1=1,MAX
      I3=I1+1
      IS=0
      DO 20 I=1,NV
      IF((KH(I1,I)*I).EQ.G(IG,1))GO TO 25
      IF((KH(I1,I)*I).NE.G(IG,2))GO TO 20
      IS=1
20    CONTINUE
      GO TO 100
25    IA=1+IS
      DO 75 I2=I3,MAXH
      DO 30 I=1,NV
      IF((KH(I2,I)*I).EQ.G(IG,IA))GO TO 35
30    CONTINUE
      GO TO 75
35    KMAX=KMAX+1
      DO 40 I=1,NV
40    H(KMAX,I)=KH(I1,I)*KH(I2,I)
      H(KMAX,G(IG,1))=1
      H(KMAX,G(IG,2))=1
75    CONTINUE
100   CONTINUE
C
C     ELIMINATE SUBSETS
C
      MAXH=KMAX
      DO 150 I=1,KMAX
      CALL SUBST1(I,NV,0)
150   CONTINUE
      MAXH=SMAXH
      DO 900 I=1,MAXH
      DO 900 J=1,NV
900   H(I,J)=KH(I,J)
1000  CONTINUE
      RETURN
      END
```

### 4.5 *RS-Procedure*

**Algorithmic scheme of the *RS*-procedure.** *Given $G_{k,i} \in S_G$ and $\rho_k = r(G_{k,i})$, to generate all immediate refinements of $G_{k,i}$ in class $S_G$ and within*

*the equivalence class characterized by graph* $\rho_k$: (1) take one particular set $E_a \in G_{k,i}$ such that $|E_a| \geqq 3$ and replace $G_{k,i}$ with $G'_{k,i} = (G_{k,i} - \{E_a\}) \cup E'_a$, where $E'_a = \{E_{a,j} | E_{a,j} \subset E_a, |E_{a,j}| = |E_a| - 1\}$; (2) for each $E_{a,j} \in E'_a$ and for each $E_b \in G_{k,i} - \{E_a\}$, if $E_{a,j} \subset E_b$, then exclude $E_{a,j}$ from $G_{k,i}$, and record the result of this step as an immediate refinement of $G_{k,i}$; (3) repeat steps (1)–(2) for all sets $E_a \in G_{k,i}$ such that $|E_a| \geqq 3$.

*Program step specifications* (Table III):

*Step* (1): In loop 1000 those subsets of variables in $H$ are considered which have a cardinality greater than or equal to input parameter $IC$ ($IC = 3$). The subset is stored in $V$. It is broken down into next smaller subsets in loop 50. Loop 40 addes the new subsets to $H$.

*Step* (2): Executed in loop 70.

*Step* (3): Variable $IH$ identifies the next subset of variables in $H$ to be broken down.

### 4.6 *RG-Procedure*

The ***RG*-procedure** can be obtained from the *RS*-procedure simply by replacing the requirement that $|E_a| \geqq 3$ in step (1) of the algorithmic scheme by a weaker requirement that $|E_a| \geqq 2$ (Ref. 4). Similar modification is made in the program for the *RS*-procedure in Table III, where the input parameter $IC$ has a value of 2, as used in statement 11.

### 4.7 *AS-Procedure*

**Algorithmic scheme of the *AS*-procedure.** *Given* $G_{k,i} \in S_G$ *and* $\rho_k = r(G_{k,i})$, *to generate all immediate aggregates of* $G_{k,i}$ *in class* $S_G$ *and within the equivalence class characterized by graph* $\rho_k$: (1) for each pair $E_a \in G_{k,i}$ and $v_\alpha \in V - E_a$, if $(v_\alpha, v_\beta) \in \rho_k$ for all $v_\beta \in E_a$, then record the pair $E_a$, $E'_a = E_a \cup \{v_\alpha\}$, and let $X$ denote the set of all recorded pairs $(E_a, E'_a)$; (2) for every two pairs $(E_a, E'_a) \in X$ and $(E_b, E'_b) \in X$, if $E'_a \supseteq E'_b$, then exclude $(E_a, E'_a)$ from $X$; (3) for each pair $(E_a, E'_a)$ left in $X$ after executing step (2): if each subset of $E'_a$ of cardinality $|E'_a| - 1$ is a subset of some $E_x \in G_{k,i} (E_x \neq E_a)$, then construct $G'_{k,i} = (G_{k,i} - Y) \cup \{E'_a\}$, where $Y = \{E_y | E_y \in G_{k,i}, E_y \subset E'_a\}$, and record $G'_{k,i}$ as an immediate aggregate of $G_{k,i}$; else, construct immediate aggregates one of each of the subsets of $E'_a$ of cardinality $|E'_a| - 1$ which is not a subset of some $E_x \in G_{k,i} (E_x \neq E_a)$ — by adding that subset to $G_{k,i}$.

TABLE III FORTRAN IV program for the *RS*-procedure.

```
      SUBROUTINE RSPROC(IC)
      ERR=1
      KMAXH=MAXH
      DO 1000 IH=1,KMAXH
      DO 10 I=1,NV
10    N=N+H(IH,I)
11    IF(N.LE.IC) GO TO 1000
      ERR=0
C
C     A COMPATIBLE CAN BE BROKEN DOWN
C
      DO 20 I=1,MAXH
      DO 20 J=1,NV
20    KH(I,J)=H(I,J)
      SMAXH=MAXH
      L=0
      DO 30 I=1,NV
      IF(H(IH,I).EQ.0) GO TO 30
      L=L+1
      V(L)=I
30    CONTINUE
      MAX=N-1
      MAXC=2**N
C
C     BREAK V IN MAX-SIZE COMPATIBLES
C
      DO 50 I=1,MAXC
      CALL BINARY (I,L,Q,SQ)
      IF(SQ.NE.MAX) GO TO 50
      MAXH=MAXH+1
      DO 40 J=1,L
      IF(Q(J).EQ.0) GO TO 40
      H(MAXH,V(J))=1
40    CONTINUE
50    CONTINUE
      DO 60 I=1,NV
60    H(IH,I)=0
      IE(2)=0
      ID=MAXH-SMAXH
      DO 70 I=1,ID
      IE(1)=MAXH+I
      CALL SUBST2(NV,IE,MV,LMV,0)
70    CONTINUE
      MAXH=SMAXH
      DO 900 I=1,MAXH
      DO 900 J=1,NV
900   H(I,J)=KH(I,J)
1000  CONTINUE
      RETURN
      END
```

The program for the *AS*-procedure, listed in Table IV, makes use of a special routine, which retrieves all possible compatibles of the given graph. As such, Steps (1)–(3) in the algorithmic scheme are largely taken care of by that subroutine. Loop 100 in the program adds to $H$ all next higher compatibles one at a time (loop 40). Subsequently, subsets of the new compatibles are eliminated in loop 50.

### 4.8 *AG-Procedure*

The ***AG*-procedure** can be obtained from the *AS*-procedure by adding to its algorithmic scheme

TABLE IV FORTRAN IV program for the *AS*-procedure.

```
      SUBROUTINE ASPROC(IRET,ERR)
      ERR=0
      IF(IRET.NE.0) GO TO 25
C
C     SAVE ORIGINAL OR MINIMAL ADJUSTED H
C
      SMAXH=MAXH
      DO 10 I=1,MAXH
      DO 10 J=1,NV
10    KH(I,J)=H(I,J)
C
C     CALL CLIQUE AND RETRIEVE FROM G
C     ALL POSSIBLE COMPATIBLES
C
      KMAXH=SMAXH
      DO 20 I=3,NV
      CALL CLIQUE(I,I,NV)
      IF(MAXH.EQ.0) GO TO 25
      DO 15 J=1,MAXH
      KMAXH=KMAXH+1
      DO 15 K=1,NV
15    KH(KMAXH,K)=H(J,K)
20    CONTINUE
25    IRET=1
      IN=0
      IF(KMAXH.EQ.SMAXH) GO TO 999
      IST=SMAXH+1
      DO 100 I=IST,KMAXH
      MAXH=1
      DO 30 J=1, NV
30    H(MAXH,J)=KH(I,J)
      DO 40 K=1,SMAXH
      MAXH=MAXH+1
      DO 40 L=1,NV
40    H(MAXH,J)=KH(K,L)
C
C     ELIMINATE SUBSETS AND IF
C     COMPATIBLE ADDED IS TOO LARGE
C     DISREGARD NEW STRUCTURE(IW=-1)
C
      DO 50 J=2,MAXH
      IW=1
      CALL SUBST1(J,NV,IW)
      IF(IW.EQ.-1) GO TO 100
50    CONTINUE
      IN=IN+1
100   CONTINUE
      IF(IN.NE.0) RETURN
999   ERR=1
      RETURN
      END
```

the following two steps: (4) include one edge into the graph $\rho_k$, say edge $(v_i, v_j)$, such that $v_i \neq v_j$ and $(v_i, v_j) \notin \rho_k$, and construct $G'_{k,i} = G_{k,i} \cup \{\{v_i, v_j\}\}$; if sets $\{v_i\}$ or $\{v_j\}$ are included in $G'_{k,i}$, then exclude them and record the result of this step as an immediate aggregate of $G_{k,i}$; (5) repeat step (4) for all edges $(v_i, v_j) \notin \rho_k$.

In addition to the subprogram for the *AS*-procedure (Table IV), the following must be inserted right after statement 10:

$$G(L, NG+2) = 1.$$

The control program determines the input parameter $L$ which indicates the edge to be added to the graph.

### 4.9 *Procedure for the Identification of Duplicates*

As described in Sections 4.1–4.4, immediate refinements or aggregates of *C*-structures and *P*-structures are basically generated by operating on the graphs of the given structures. This is possible because each graph uniquely represents one *C*-structure and one *P*-structure. Each immediate refinement is generated by excluding one edge from a given graph; each immediate aggregate is generated by adding one edge to the graph.

It is rather obvious that given two graphs, say $\rho_1$ and $\rho_2$, representing two *C*-structures or two *P*-structures, the same graph can be obtained by excluding (or adding) one edge to $\rho_1$ and one edge to $\rho_2$ if and only if (i) $\rho_1$ and $\rho_2$ have the same number of edges, and (ii) $\rho_1$ and $\rho_2$ are different in exactly two edges

When condition (i) is satisfied, the structures represented by graphs $\rho_1$ and $\rho_2$ are said to be located at the same level of refinement. Since the search through the lattice of *C*-structures or *P*-structures always proceeds by levels of refinements, i.e., "successful" structures at each level are further refined or aggregated, condition (i) is always satisfied. Condition (ii) has to be tested and is the key for the identification of duplicates, as described by the following algorithmic scheme.

**Algorithmic scheme of the procedure for the identification of duplicates** generated by the refining procedures (or aggregating procedures, respectively) for *C*-structures or *P*-structures. *Given graphs $\rho_1, \rho_2, \ldots, \rho_q$ with the same number of edges and defined on the same set V, to identify edges in graphs $\rho_k$ $(k=2,3,\ldots,q)$ whose exclusion (or inclusion, respectively) would produce duplicates of graphs obtained from $\rho_i$, $i<k$:* (1) Let $k=2$; (2) let $i=1$ and $D_k=\varnothing$; (3) determine $X_{k,i}=\rho_k - \rho_k \cap \rho_i$ (or $X_{k,i}=\rho_i - \rho_k \cap \rho_i$, respectively); (4) if $|X_{k,i}|=1$, make $D_k \to D_k \cup X_{k,i}$; (5) if $i<k-1$, make $i \to i+1$ and go to (3); (6) $D_k$ is the set of all edges in $\rho_k$ whose exclusion (or inclusion, respectively) would generate duplicates; (7) if $k<q$, make $k \to k+1$ and go to (2); otherwise stop.

*Program step specifications for the identification of duplicates of C-structures and P-structures* (Table V):

*Step* (1): $K$ is controlled from outside of the program.

*Step* (2): Executed in loop 50 with $I$ varying from 1 to $K-1$.

*Step* (3): Executed within loop 20.

*Step* (4): Executed by loops 30 to 40.

*Step* (5): Variable $I$ is incremented in loop 50.

*Step* (6): After loop 50 is completed, the result $(D_k)$ is returned to the control program.

*Step* (7): The current value of $K$ is returned to the control program.

Note: Variable name *ISW* contains a code, which determines which of the two options in Step (3) are to be executed; 0 and 1 are used, respectively, for the refinement and aggregating procedures.

*Example.* Let six $C$-structures $C_k$ or $P$-structures $P_k$ $(k=1,2,\ldots,6)$ of four variables and at the third level of refinement be given in terms of graphs $\rho_k = r_c(C_k)$ or $\rho_k = r_p(P_k)$, respectively. Let the graphs be defined by matrix

$$G = \begin{array}{c} \begin{matrix} & \rho_1 & \rho_2 & \rho_3 & \rho_4 & \rho_5 & \rho_6 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 & 0 & 1 & 1 \\ 2 & 4 & 0 & 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

which has exactly the same meaning as matrix $G$ in the FORTRAN program. When the procedure for the identification of duplicates generated by the refining procedures is used, the following matrix results

$$D = \begin{array}{c} \begin{matrix} & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 & 0 & 1 & 1 \\ 2 & 4 & 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}.$$

Column $D_k (k=1,2,\ldots,6)$ in matrix $D$ specifies the set of edges in $\rho_k (k=1,2,\ldots,6)$ whose exclusion would generate duplicates. For example, let $k=3$ be defined in step (7) so that the procedure will operate now on graph $\rho_3$ by executing the following steps: (2) $i=1$, $D_3=\varnothing$; (3) $X_{3,1}=\{(2,4)\}$; (4) $D_3=\{(2,4)\}$; (5) $i=2$; (3) $X_{3,2}=\{(1,4)\}$; (4) $D_3=\{(2,4),\ (1,4)\}$; (5) $i<k-1$ is false; (6) $D_3=\{(2,4),\ (1,4)\}$ is the set of all edges in $\rho_3$ whose exclusion would generate duplicates; (7) $k=4$; (2) $i=1$, $D_4=\varnothing\ldots$: the procedure would now operate on graph $\rho_4$, etc.

When the procedure is used for the identification of duplicates generated by the aggregating procedures, the result is the following matrix:

$$D = \begin{array}{c} \begin{matrix} & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 4 & 0 & 1 & 0 & 0 & 0 & 1 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

As far as the identification of duplicates of $G$-structures is concerned, which is needed for procedures *RS*, *AS*, *RG* and *AG*, no sufficiently simple algorithm seems to exist which would identify the duplicates before they are generated. Hence, the identification of duplicates of $G$-structures is entered after the new $G$-structures (immediate refinements or aggregates) are generated. Each generated $G$-structure is simply checked, immediately after its generation, for equality with each of the previously recorded $G$-structures at the same level of refinement. When the equality with a previous $G$-structure is found, the tested $G$-structure is discarded as a duplicate. This algorithm is rather obvious and we do not deem it necessary to list the corresponding program here.

### *4.10 M-Procedures*

In this section, a new pair of refining and aggregating procedures for generating reconstruction hypotheses is introduced. They are based on the $M$-structures and, to some extent, the $I$-structures. As already mentioned, they are viewed as "semi-global" in the sense that they perform a

TABLE V FORTRAN IV program for the procedure of identifying duplicates of *C*-structures or *P*-structures.

```
      SUBROUTINE DUPLIC(K,T)
      INTEGER T(105),X(105),G(105,17)
C
C     ISW IS INDICATOR (0=REFIN/1=AGGREG)
C     T IS TRUTH VECTOR, INITIALLY = 1
C     (K=NG FOR OTHER PROCEDURES)
C
      ISW=T(1)
      DO 10 I=1,MAXG
10    T(I)=1
C
C     LOOP 50 RUNS FROM I=1 TO K-1
C     X IS COMPLEMENT INTERSECTION FOR K
C
      MAX=K-1
      DO 50 I=1,MAX
      DO 20 J=1,MAXG
      X(J)=G(J,2+I)*G(J,2+K)
      IF(ISW.EQ.0)X(J)=G(J,2+K)-X(J)
      IF(ISW.EQ.1)X(J)=G(J,2+I)-X(J)
20    CONTINUE
      N=0
      DO 30 L=1,MAXG
30    N=N+X(L)
      IF(N.GT.1) GO TO 50
      DO 40 L=1,MAXG
      T(L)=T(L)-X(L)
      IF(T(L).LT.0)T(L)=0
40    CONTINUE
C
C     T(L)=0 WHEN PAIR L IN G IS NOT
C     TO BE CONSIDERED FOR
C     AGGREGATION OR REFINEMENT.
C
50    CONTINUE
      RETURN
      END
```

global search in the individual equivalence classes in $S_G/r$.

Let $M_{i,k}$ denote an $M$-structure which is in the equivalence class represented by a $C$-structure $C_i$. Index $i$ identifies the equivalence class; index $k$ distinguishes different $M$-structures in the same equivalence class. Let $S_{M,i}$ denote the set of all $M$-structures in the equivalence class represented by $C_i$, i.e., $S_{M,i}=\{M_{i,k} | M_{i,k}\in S_M, r(M_{i,k})=r(C_i), C_i\in S_C\}$. Let function

$$r_M: S_M \to R$$

be defined in the same way as function $r$ but with the domain restricted to $S_M$.

With this inventory of symbols, we can now formulate a rather obvious refinement procedure for $M$-structures or *RM-procedure.*

**Algorithmic scheme of the RM-procedure.** *Given* $M_{i,k}\in S_{M,i}$ *(initially* $M_{i,0}=C_i$*) and graph* $\rho_i=r_C(C_i)=r_M(M_{i,k})$*, to determine all immediate refinements of* $M_{i,k}$ *in set* $S_{M,i}$: (1) for each $E_a\in M_{i,k}$, if $r_M(M_{i,k}-\{E_a\})=r_M(M_{i,k})$, then record $M'_{i,k}=M_{i,k}-\{E_a\}$ as an immediate refinement of $M_{i,k}$ in set $S_{M,i}$; (2) repeat step (1) for all $E_a\in M_{i,k}$.

For the formulation of an aggregating procedure for $M$-structures, or *AM-procedure,* we need the following notion of irredundant $M$-structures or $I$-structures:[4] $M_{i,k}$ is said to be irredundant $(M_{i,k}=I_{i,j}, I_{i,j}\in S_I)$ if and only if there is no $\alpha\neq k$ such that $M_{i,\alpha}\leqq M_{i,k}$.

**Algorithmic scheme of the AM-procedure.** *Given* $M_{i,k}\in S_{M,i}$ *(initially* $M_{i,k}=I_{i,j}$*) and* $C_i$*, to determine all immediate aggregates of* $M_{i,k}$ *in set* $S_{M,i}$: (i) make $M'_{i,k}=M_{i,k}\cup\{E_a\}$, where $E_a\in C_i-M_{i,k}$, and record $M'_{i,k}$ as an immediate aggregate; (2) repeat step (1) for all $E_a\in C_i-M_{i,k}$.

The identification of duplicates in the *RM*-procedure and *AM*-procedure can be accomplished by the following trivial modification of the procedure for the identification of duplicates of $C$-structures or $P$-structures (Section 4.9): replace "graphs $\rho_1, \rho_2, \ldots, \rho_q$" by "hypergraphs $M_{i,1}, M_{i,2}, \ldots, M_{i,q}$."

All the described procedures for $M$-structures are rather simple and straightforward, and we do not deem it necessary to list their programs here.

For small number of variables ($n\leqq 5$), the application of the $M$-procedures is vacuous as there are no redundant maximal compatibles in the graphs. They become applicable when $n=6$, and their significance rapidly escalates with increasing number of variables.

*Example.* Let $C$-structure $C_i$, which can also be viewed as $M$-structure $M_{i,0}$ (i.e., $M_{i,0}=C_i$), be given in terms of graph $\rho_i=r_C(C_i)$ specified by the diagram in Figure 5 (variables are denoted by integers $1,2,\ldots,6$). It can be seen by simple inspection of the diagram that the graph is associated with the following eight maximal compatibles:

$$A=\{1,2,3\} \qquad E=\{1,5,6\}$$
$$B=\{2,3,4\} \qquad F=\{1,2,6\}$$
$$C=\{3,4,5\} \qquad G=\{1,3,5\}$$
$$D=\{4,5,6\} \qquad H=\{2,4,6\}$$

The *RM*-procedure, when applied to structure $C_i=M_{i,0}$ would generate eight $M$-structures $M_{i,k}$ $(k=1,2,\ldots,8)$ of the type $M_{i,k}=C_i-X_k$, where $X_k$ is specified in Table VI. When applied again to each of these $M$-structures, the *RM*-procedure

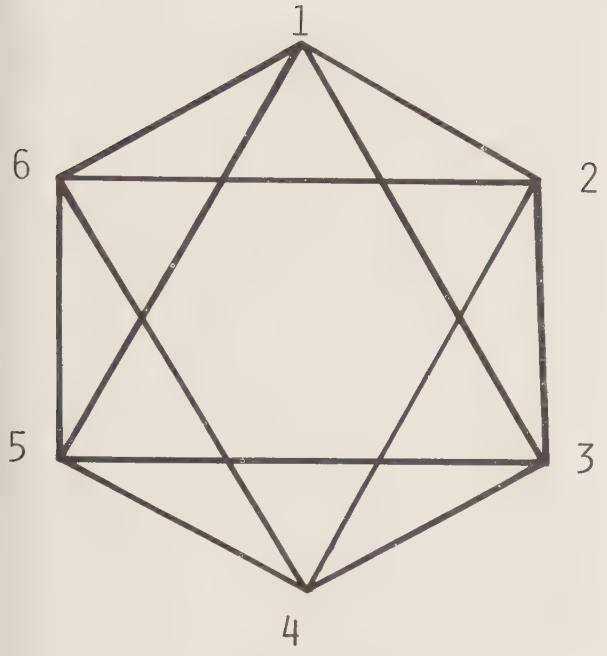


FIGURE 5 Illustration of the $M$-procedures.

would generate appropriate $M$-structures in the set $\{M_{i,k} | k = 9, 10, \ldots, 24\}$, as indicated in Table VI. Some of these structures (those with $k = 10, 14, 17, 24$) are irredundant and, consequently, do not have any refinements in the equivalence class characterized by structure $C_i$. Other structures in the set can further be refined in the way which is fully described in Table VI. The lattice $(S_{M,i}, \leqq)$ defined by Table VI contains five levels of refinement (including the zero level of $M_{i,0}$).

TABLE VI

Lattice $(S_{M,i}, \leqq)$ of $M$-structures in the equivalence class characterized by $C$-structures $C_i$ specified by the graph $r_t(C_i)$ in Figure 5.

| $k$ | $X_k$ | Immediate refinements | Immediate aggregates | Irredundant | Level of refinement |
|---|---|---|---|---|---|
| 0 | $\varnothing$ | 1, 2, ..., 8 | none | No. | 0 |
| 1 | $\{A\}$ | 9, 10, 11, 12 | 0 | No | 1 |
| 2 | $\{B\}$ | 13, 14, 15, 16 | 0 | No | |
| 3 | $\{C\}$ | 9, 17, 18, 19 | 0 | No | |
| 4 | $\{D\}$ | 10, 13, 20, 21 | 0 | No | |
| 5 | $\{E\}$ | 11, 14, 18, 22 | 0 | No | |
| 6 | $\{F\}$ | 15, 17, 20, 23 | 0 | No | |
| 7 | $\{G\}$ | 16, 21, 23, 24 | 0 | No | |
| 8 | $\{H\}$ | 12, 19, 22, 24 | 0 | No | |
| 9 | $\{A, C\}$ | 25, 26 | 1, 3 | No | 2 |
| 10 | $\{A, D\}$ | none | 1, 4 | Yes | |
| 11 | $\{A, E\}$ | 25, 27 | 1, 5 | No | |
| 12 | $\{A, H\}$ | 26, 27 | 1, 8 | No | |
| 13 | $\{B, D\}$ | 29, 30 | 2, 4 | No | |
| 14 | $\{B, E\}$ | none | 2, 5 | Yes | |
| 15 | $\{B, F\}$ | 29, 31 | 2, 6 | No | |
| 16 | $\{B, G\}$ | 30, 31 | 2, 7 | No | |
| 17 | $\{C, E\}$ | none | 3, 6 | Yes | |
| 18 | $\{C, E\}$ | 25, 28 | 3, 5 | No | |
| 19 | $\{C, H\}$ | 26, 28 | 3, 8 | No | |
| 20 | $\{D, F\}$ | 29, 32 | 4, 6 | No | |
| 21 | $\{D, G\}$ | 30, 32 | 4, 7 | No | |
| 22 | $\{E, H\}$ | 27, 28 | 5, 8 | No | |
| 23 | $\{F, G\}$ | 31, 32 | 6, 7 | No | |
| 24 | $\{G, H\}$ | none | 7, 8 | Yes | |
| 25 | $\{A, C, E\}$ | 33 | 9, 11, 18 | No | 3 |
| 26 | $\{A, C, H\}$ | 33 | 9, 12, 19 | No | |
| 27 | $\{A, E, H\}$ | 33 | 11, 12, 22 | No | |
| 28 | $\{C, E, H\}$ | 33 | 18, 19, 22 | No | |
| 29 | $\{B, D, F\}$ | 34 | 13, 15, 20 | No | |
| 30 | $\{B, D, G\}$ | 34 | 13, 16, 21 | No | |
| 31 | $\{B, F, G\}$ | 34 | 15, 16, 23 | No | |
| 32 | $\{D, F, G\}$ | 34 | 20, 21, 23 | No | |
| 33 | $\{A, C, E, H\}$ | none | 25, 26, 27, 28 | Yes | 4 |
| 34 | $\{B, D, F, G\}$ | none | 29, 30, 31, 32 | Yes | |

## 5 SUMMARY AND GENERAL DISCUSSION

A specialized version of the reconstructability analysis, as understood in this paper, was first suggested by Klir in 1976, under the name "structure identification".[8] A general framework for the reconstructability analysis, and particularly the problem of generating reconstruction hypotheses, which was recently formulated by Cavallo and Klir,[4] is fully adopted in this paper. Algorithmic schemes of the various procedures for generating reconstruction hypotheses, derived in the previous paper,[4] are elaborated in this paper up to their formal definitions in FORTRAN IV.

All procedures described in this paper are based on the ordering of structure systems by the refinement relation. From one point of view, the procedures can be classified into those which generate all immediate refinements of a given structure in some specified set of structures, those which generate all immediate aggregates in the specified set, and those which prevent the generation of duplicate structures.

From another point of view, the procedures can be classified into global, semi-global and local procedures, depending on the set of structures within which the generation takes place. Procedures for $C$-structures or $P$-structures are global in the sense that they operate only on suitable equivalence classes of all possible reconstruction hypotheses. Procedures for $M$-structures are semi-global in the sense that they search only through convenient representative reconstruction hypotheses ($M$-structures) in each of the equivalence classes. Procedures for $G$-structures are local

in the sense that they search locally through all possible reconstruction hypotheses.

While in the refining procedures subsystems are broken down into smaller subsystems, the aggregating procedures require that larger subsystems be constructed from smaller ones. Since it is easier to break down a system than to construct one, the aggregating procedures are more complex than the refining procedures. Hence, the refining procedures are preferable in all instances except those where an aggregating procedure is necessary or desirable for some non-computational reasons.

Insufficient developments in the theory of hypergraphs[1] cause at this time the main difficulty in determining estimates of computational complexities of the various procedures presented in this paper. To determine theoretical estimates of these complexities, one would need formulas (or reasonable upper bounds) for the average (or maximum) number of edges in the hypergraphs which qualify as *G*-structures and those which qualify as *C*-structures, given a particular number of variables and a number of edges in the corresponding graphs. Until such formulas (or upper bounds) become available, we shall be limited to experimentally determined estimates. Investigations in this respect are currently under way and will be combined with similar investigations for the evaluating procedures and decision making procedures of the reconstructability analysis. This will hopefully provide some insight of our (i.e., human beings equipped with computers) principal limitations in solving the general reconstructability problem.

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*For a biography and photograph of George Klir, please see Volume 5, No. 3, p. 171 of this journal.*

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# *Letters to the Editor*

## ON COMPLEMENTARY ASPECTS FOR GENERAL SYSTEMS

Sir—In his lucid answer to Donnelly, Pattee[5] clarifies the problem of complementarity for description of linguistic constraints and description of dynamical laws in terms of physical arguments with reference to Bohr's conceptions of complementarity.

Pattee's main point is that general systems theories cannot be expected to provide adequate models of systems, which function through their own internal descriptions, until the epistemological problem of complementarity between subject and object is more clearly recognized.

I fully agree with this point, as well as with strengthening it by recognizing complementarity in a wider logical context (still much in the spirit of Bohr).

It is important then to clarify Pattee's statement that "complete self-description of any system is a physical impossibility, though not a logical or formal impossibility".

If complete self-description is meant to describe all of the description process itself (including both description and interpretation), then it is *logically impossible*. This has been shown by Tarski[6], and somewhat later by Gödel[1].

However, Tarski's methodology suggests that rather embracing forms of self-description are possible (cf. Refs. 2, 3), indeed such strong forms that they have been called "complete" although that word be better reserved for the above sense. Similarly some self-referring function spaces have been called "type-free" although logical types are needed for their description.

It may be less well-known that Tarski's result was anticipated in a conjecture by Russell. In his introduction to Wittgenstein's Tractatus[17], Russell writes:

> These difficulties suggest to my mind some such possibility as this: that every language has, as Mr Wittgenstein says, a structure concerning which, *in the language*, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit.

In the light of Tarski's result [6], this conjecture concerning an extension of the theory of types appears quite insightful. As a consequence, the descriptions and interpretations which constitute a language are *complementary* in the sense that not all of the interpretations can be described as sentences in the language itself.

Since many interesting systems are describing, or can be regarded linguistic (cf. Refs. 3, 4) this logical notion of complementarity will have important consequences for the development of general systems theories.

Furthermore, we may understand why studies of complementarity, or studies of hierarchy theory, must remain intrinsically difficult. Namely, because the hierarchy itself cannot be fully described, nor the complementarity for description and interpretation. In this connection Russell continues his above quotation as follows:

> Mr Wittgenstein would of course reply that his whole theory is applicable unchanged to the totality of such languages. The only retort would be to deny that there is any such totality.

Today, with our clearer understanding of undecidability and nondescribability (due to Gödel and Tarski), we would prefer to say that such a totality of languages (such a heirarchy of languages, such a complementarity) cannot be completely described.

I think it is important to stress that these apparently negative results do have an underlying positive nature. Surely the results suggest bounds, but not of such a nature that human knowledge would progress to a certain boundary and then no further. Rather, the productive nature behind the results of Tarski are suggestive for how to proceed in our studies of hierarchies, and of complementarities, for how long as we please to steadily discover new results.

I fully agree Pattee on the importance of general systems studies emphasizing a wide linguistic perspective recognizing concepts like for example complementary and linguistic hierarchy.

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## IDENTIFICATION OF STOCHASTIC SYSTEMS

Sir—In the treatments of the above topic by Gaines† and by Witten‡ there is a small but important point which is, by one of them, acknowledged but quickly passed over and, by the other, ignored. Their identification procedures operate in two stages. In the first stage an automaton structure is produced, and in the second the transition probabilities between states are estimated. The point which is glossed over is that a stochastic system may be such that the model of it produced in these two stages is automatically inferior to one which could be produced by taking account of probabilities from the start.

To illustrate this simple point, consider a system which is accurately represented by the stochastic automaton shown, in Mealy form, in Figure 1. A modelling procedure which takes account only of occurrence and non-occurrence of transitions, and which operates in terms of the Mealy representation, would produce the network of Figure 2. This can only lead to a poor approximation.

A two-stage procedure operating in terms of the Moore representation would model this particular system accurately by forming the network of Figure 3, but it is easy to devise slightly more complex systems which would not be satisfactorily modelled by this technique either.

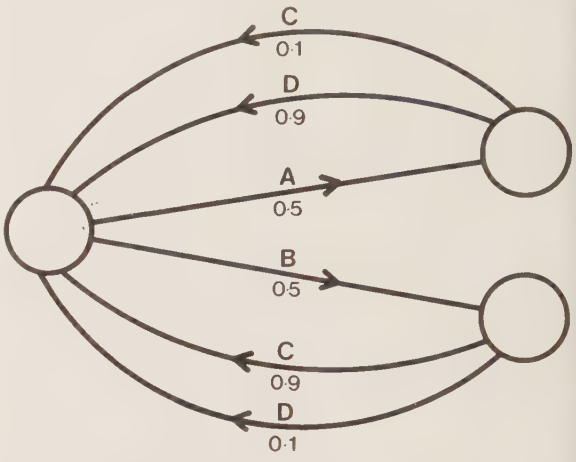


FIGURE 1 True structure of system.

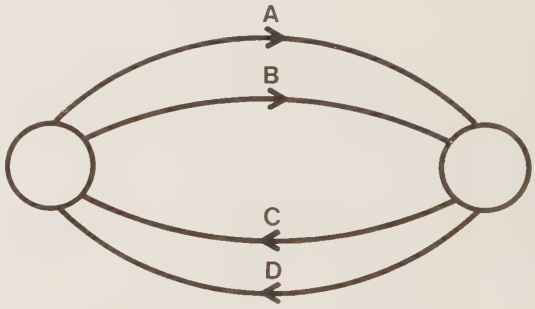


FIGURE 2 Mealy model.

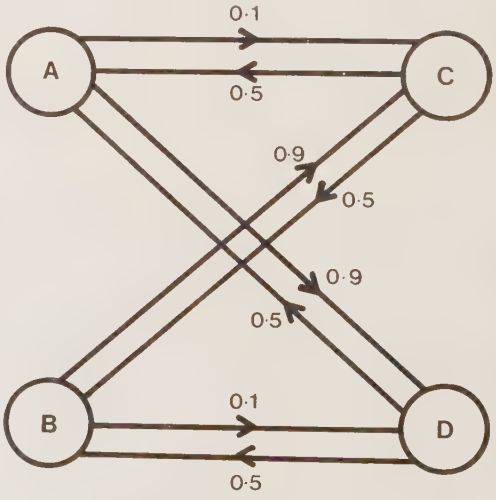


FIGURE 3 Moore model.

†Brian R. Gaines, "System identification, approximation and complexity". *Int. J. General Systems*, **3**, pp. 145–174, 1977.

‡Ian H. Witten, "Approximate, non-deterministic modelling of behaviour sequences", *Int. J. General Systems*, **5**, pp. 1 12, 1979.

Probably these workers can defend the use of two-stage procedures on the basis of their experience of practical applications. It is possible that systems which cannot be accurately modelled using their techniques are rare in the natural world. Nevertheless the point merits some attention.

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# *Book Reviews*

*The editor invites books for review on any subject relevant to general systems. Books for review must be sent to Dr. George Klir, Editor,* International Journal of General Systems, *Department of Systems Science, School of Advanced Technology, State University of New York, Binghamton, New York 13901, U.S.A.* (*and* not *to the publisher*).

## FUNDAMENTALS OF MEASUREMENT AND REPRESENTATION OF NATURAL SYSTEMS, by Robert Rosen. North-Holland, New York, 1978, 221 pp.

This book formulates, and studies within its formulation, some fundamental issues of the theory of modelling. The approach is to employ the systems concepts of state and state transition, the algebraic apparatus of partitions, congruence relations and groups, and sometimes, the language of category theory to formalize the role of observation in modelling. The latter is fundamental since the observer's measurement modes determine his domain of possible models and their properties such as variable linkages, decomposability, symmetries and similarities.

This is the kind of book which should be read by every scientist and applied mathematician for the perspective it gives on the fundamentals of their trade. Unfortunately, the mathematics, and especially the language of partitions, requires quite some visualization abilities, which will make for tough going for those unfamiliar with such concepts. This, despite the author's fine explanatory style. Nevertheless, even the cognoscenti would have been helped by the provision of many more illustrations and simple examples. Still it must be acknowledged that the freeing of the mind afforded by abstraction can be purchased only at the price of some effort.

Since there will be other explanatory reviews of this book, and because of its foundational character, the following will be less a review than a critique. In particular, since the author does not do it himself, it is important to place this work within the systems and modelling theory literature. This perspective helps to understand how the work is similar to, and yet different from, other contemporary systems approaches. Next I want to take issue with the author's arguments, though not necessarily his conclusions, against reductionism. Finally, there are a few technical errors which should be pointed out.

In contrast to the brunt of systems formalization, which emerged from an engineering origin, Rosen's departure point is the quantum mechanics of physics. Indeed, the latter theory provides, by far, the most accepted systems modelling paradigm of contemporary science. This book can be seen as Rosen's attempt to free himself, and the rest of us, from its universal, but restrictive, claims. Rosen points out the "gratuitous, non-physical assumptions" that have crept into the foundations of quantum theory, which he states "are responsible for its universal character". An example is this assumption of state variable set uniqueness. It seems to me that here Rosen is successful in undermining the *a priori* basis for believing that quantum theory will ultimately provide the ground models for all aspects of nature (but it would be interesting to get a quantum theorist's reaction). Yet even if Rosen opens up the possibility that not everything can be explained by physics, he does not prove the stronger case that it is *impossible* for physics to give such explanations. To this we return momentarily.

Rosen's formalism—quantum theory cleansed of gratuitous assumptions—is familiar looking to general systems researchers. His "system" is essentially a deterministic, time invariant, autonomous transition system (cf. Wymore (1973), Padulo and Arbib (1976), etc.). In a departure from physics, Rosen insists that all observables (output maps) be measurable (linked to meters). But his notion of state variable set, derived from physics, differs from the usual systems concept. For Rosen it represents a minimally redundant subset of the set of observables. The usual systems concept relates to the minimal information required to predict all future output trajectories. Thus in the usual concept, a state variable set's being so depends on both the dynamics and the

observables (see Zeigler, 1976), while in Rosen's version, only the latter is the determinant. The author's attempt to reconcile the two definitions (on page 87) only serves to confuse the issue.

There is a more profound methodological problem in Rosen's formalism. Since he does not recognize input ports, and a clear separation of system from environment, he is led to employ such concepts as "imposing dynamics" on a system which, picturesque as they are, are never formally defined. This vagueness could be cleared up by adopting the more usual input-state-output construction and would remove the somewhat mystical atmosphere prevailing in the reductionism discussion to which we now turn.

Rosen asserts that the only valid reduction of a system is its decomposition into, what he calls "natural subsystems", i.e. an irreducible parallel decomposition (cf. Hartmannis and Stearns, 1965). This is so since each component is independently isolatable and it is a simple matter to reconstruct the original dynamics as a cross-product of the component dynamics. Other decompositions require an "imposition of dynamics" and "breaking of linkages" to isolate their components. Rosen implies that the effects of this imposition are always irreversible, thereby preventing the reconstruction of the original dynamics from those of its components. Since it is evident to Rosen that physical particles—atoms, molecules, etc.—are not "natural subsystems", reductionism fails.

But is it true that only parallel decompositions qualify as valid reductions? Consider a radio, a particular existing physical object. True enough, to decompose it into resistors, capacitors, inductors and transistors, you must "impose a dynamics" not usually associated with the radio—i.e., you must cut the wires connecting these components. True enough, these components do not (usually) form a parallel decomposition and the radio dynamics is not a cross-product of the component dynamics. But there can be no doubt that such components constitute a *useful* decomposition—there could be no radios without a knowledge of their dynamics and how to couple them to realize the radio dynamics. Indeed, the last sentence is the key—a valid decomposition requires that both a set of components and their coupling recipe be given such that the resulting network simulates the original system. Were we to have the knowledge of how to couple physical particles together to realize an organism, no one would hesitate to call this a valid reduction of biology to physics.

To retain viability, Rosen's argument against reductionism must thus be interpreted as the claim that we can never find out the true coupling recipe for reconstructing organisms from particles. Presumably this is because in that act of "fractionation" (decomposition) we do irreversible damage by the "breaking of linkages". That we can cut the wires of a radio and then reconnect them shows that fractionation is not irreversible in general. Indeed, work in automata shows that reversible decomposition can be done in principle. It may be noted that this work actually employs the partition formalism in greater depth (see Zeigler, 1976, Chapters 12, 15 for a review). This is not to say that a valid decomposition into particles is easy or even empirically possible. Just that Rosen's argument, based on a misconstrued reductionism, does not provide an *a priori* basis for these beliefs.

Yet it is also clear that knowledge about how a radio works also lies at higher levels of abstraction, for example, in terms of functional decomposition into tuners, detectors, amplifiers, etc. Indeed, it is being increasingly recognized that there may be many (perhaps interrelated) levels of such decompositions (reducible, non-parallel) each of which constitutes a valid model and no one more intrinsically fundamental than any other (Zeigler, 1971, 1979). Thus reductionism may, in the final analysis, prove less to be impossible, than to be irrelevant to the questions of biological interest.

In reading the book I came across some passages which were confusing, some of which turned out to be lapses in logic. The following is my recollection of them.

p. 35: The construction given for extracting a partition from a cover is incorrect. It does not guarantee disjoint sets.

p. 51: It does not follow that two meters which are equivalent are homomorphically related as claimed (for one thing, they need not follow related trajectories to reach identical attractors).

p. 63: Theorem 3.3.1 is obviously false. Consider for example, the case where $h$ is a constant function.

p. 71: The concept of invisibility employed is stronger than that justified by the definition

of meter (only asymptotic states must remain unperturbed).

.72: The first sentence is true only if $R_f$ coincides with the trajectory partition of $\overline{T}_t$.

.81: The identity dynamics is *not* the only dynamics compatible with the identity relation. In fact, *every* dynamics is compatible with the identity relation.

In summary, as indicated, this has been a ritique rather than a review. The reader will find nany stimulating ideals of a fundamental charac-er not mentioned here. Responding critically, as vell as creatively, to them will prove rewarding.

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# FUZZY sets and systems


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## Aims and Scope:

The primary purpose of the INTERNATIONAL JOURNAL FOR FUZZY SETS AND SYSTEMS is to serve as a means of *professional communication between scientists and practitioners* being interested, doing research or applying Fuzzy Sets.

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**061579.1**
APPLICATION OF GENERAL SYSTEMS THEORY AND CYBERNETICS TO THE MANAGEMENT OF RESEARCH AND DEVELOPMENT, by P. D. Fosko and J. K. Hancock. 19pp. ($1.50).

The purpose of this report is to review the major contributions general systems theory and cybernetics have made to the R&D program management. Because of the holistic approach of this methodology, such contributions are often indirect and can be carried over from more general models of the management process. Accordingly we first review those systems aspects of management planning, organizing, controlling and technology forecasting that strongly impact R&D in the indirect sense. This is followed by areas of systems research, particularly in cybernetics, that have been directly concerned with technology and innovation.

**061579.2**
APPLICATIONS OF SYSTEMS THEORY TO ECOLOGY, by W. A. Johnson, *Head, Facility Support Unit, Tractor Incorporated, Rockville, Maryland; and* P. Shanker, *Manager, Applied Mechanics Section, Fairchild Space & Electronics Company, Germantown, Maryland*, 28pp. ($2.00).

This report describes some applications of the General Systems Theory to systems in ecology. This area of endeavor is still in the developing stages and several emerging characteristics of Ecosystems are presented. Specific examples of the General Systems Theory applications to various ecological systems are included to illustrate methods of analysis and modelling techniques.

A discussion of the trend to date, and where it could lead in the near future, is presented in the concluding section.

**062079.1**
DEVELOPING THE LEARNING RELATIONSHIP: IN THEORY AND PRACTICE, by C. Slawski, *California State University, Long Beach, CA*, 30pp. ($2.00).

A systems model of the learning relationship is elaborated in terms of eleven global concepts and twelve main hypotheses. The primary overall aim is to show how the numerous factors influencing one's learning or teaching experiences can be taken into account in a systematic way. The specific purpose is to illuminate the process in a way that will be useful to researchers, scholars and students in need of educational guidance. The model provides a cognitive map as well as an accounting scheme to help sort out an area of research that is too heavily overladen with unsupported opinion, and too complex for easy unraveling, let alone too difficult to put into practice consistently without deep philosophical roots supporting the practitioner.

**062079.2**
IDENTITY BARGAINING: A POLICY SYSTEMS RESEARCH MODEL OF CAREER DEVELOPMENT, by C. Slawski, *California State University, Long Beach, CA*, 45pp. ($3.00).

A detailed, general and comprehensive accounting scheme is presented, consisting of nine stages of career development, three major sets of elements contributing to career choice (in terms of personal, cultural and situational roles), and twenty hypotheses relating the separate elements. Implicit in the

model is a novel procedure and method for systematically analyzing biographies and life histories in a way that will not only help to codify the data but will also shed light on the sociological and social psychological factors underlying such lives. The aim of this approach is to facilitate understanding and research on persons in social context with a view to promoting more serious, more informed decisions about their lives, decisions made not only by administrators of government and personnel managers, but by the persons themselves.

**062079.3**
THE INTERPERSONAL CONFLICT EPISODE: A SYSTEMS MODEL, by C. Slawski, *California State University, Long Beach, CA*, 23pp. ($1.50).

A detailed systems diagram elaborates the process of dealing with a single conflict episode between two parties or persons. Hypotheses are fully stated to lead the reader through the flow diagram. A concrete example illustrates its use. Even greater detail is provided in an accounting scheme of virtually all possible variables that one might want to consider in analyzing a conflict episode. Finally, some general guidelines for applying the model are provided. The aim is to advance social scientific theory of conflict as well as to provide counselors and people in conflict with a heuristic and practical device for understanding it and thus better coping with it.

**062079.4**
INSTITUTIONALIZING INNOVATION IN AN ORGANIZATION: A MODEL AND CASE STUDY, by C. Slawski, *California State University, Long Beach, CA*, 61pp. ($5.00).

A policy systems theoretical analysis of the problem of institutionalizing innovation in an organization is summarized in a flow diagram. The model is presented in terms of specific hypotheses, and then illustrated with a case of frustrated innovation, the 1968-69 crises at San Francisco State College. The model is set up (1) to help understand the process of innovation in an organization both in theory and practice, (2) to highlight turning points and stumbling blocks in attempts to innovate, and (3) to suggest guidelines for implementing change and managing crises along the way. The result should be of benefit to theorists, researchers, policy-makers and administrators.

**062079.5**
LOVE, POWER AND CONFLICT: A SYSTEMS MODEL OF INTERPARTY NEGOTIATION, by C. Slawski, *California State University, Long Beach, CA*, 49pp. ($3.50).

Some very basic hypotheses and corresponding definitions of concepts are presented in an effort to succinctly state the relationship between three of the concepts which are among the most basic to social science, namely, love, power and conflict. This novel theory is an example of limited reduction. However, it is cast in such a way as to be applicable to both micro and macro levels of analysis, to relations between two persons or two nations. A flow diagram helps to summarize and visualize the theory as a whole. In addition, an accounting scheme and a procedure are presented for content analyzing a wide variety of materials in terms of this model. Finally a partial application will illustrate the procedure and some of the most basic hypotheses as they are used to analyze a theatre play about interpersonal conflict (David and Lisa) (Space does not permit the use of a macro level example here.) Recommendations are then made on how to promote love and avoid harmful conflict.

**062079.6**
MAPPING JOINT ACTION, by C. Slawski *Clifornia State University, Long Beach, CA*, 9pp ($1.00).

Carefully planned flow diagrams can provide an invaluable cognitive map to aid in synthesizing diverse areas of knowledge in a special brand of field theory. An unusually powerful general model is presented here in a way which is arranged to highlight the domains of structural functionalism (with the concepts of cultural, personal and situational or societal systems and roles, as well as Parson's four-functional paradigm and Merton's manifest and latent functions) and symbolic interactionism (with the concepts of self, sentiments, role taking, strategies of action, joint action, and the extensions into role making, knowledge of other, role bargaining and the I-Me interaction process). This provides an accounting scheme or extending check list for analyzing virtually any new problem that can be viewed in terms of some combination of these variables. It emphasizes process without neglecting structure. Most theories emphasize one almost to the exclusion of the other. They are by that measure incomplete as deductive theories strictly construed. The statement of specific hypotheses relating pairs of variables (contained in the flow diagram) is essential to prediction as well as to formal theorizing, and thus to understanding and explanation. The basic diagram starts with conflict between two parties, thus emphasizing relationship over individual psychology and the most universal problem of social science, how to handle conflict as a manageable problem or as a stimulus to achieving orderly social change. We thus have a prototypical example of the use of general systems theory.

**062079.7**
SOCIAL ACTUALIZATION: A THEORY OF COMMUNITY LIVING ARRANGEMENTS AND A PRACTICAL UTOPIA, by C. Slawski *California State University, Long Beach, CA*, 36pp ($3.00).

A social theory is presented about the causes and consequences of the good society. The concepts of the theory center around the goals of the greater good of the group without sacrificing basic self actualization. Preference is given to deep interpersonal and intergroup relationships rather than productivity per se. A paradigm or an accounting scheme is presented in the appendix in order to help begin a systematic evaluation of the characteristics and relative success of intentional communities of past and present. The long-range aim here is to seek out the qualities that are necessary before certain kinds of communities can become successful in terms of survival as well as fulfillment of the members.

**062079.8**
A SYSTEMS MODEL OF NEGOTIATION: ARABS VERSUS ISRAELI, by C. Slawski, *California State University, Long Beach, CA*, 53pp. ($4.50).

Application is made of a general systems model of interparty negotiation based on the interplay of hypotheses relating the key concepts of love (with its five elements of empathy, desire of the presence of the other, sharing, sacrifice and joy), as well as power and conflict in their various forms and manifestations. The macro-level case of international conflict between Arabs and Israeli since 1947 is taken here primarily because of its practical importance for world peace, and secondly because of the theoretical possibilities it offers for extending the use of this model of negotiation for synthesizing numerous fragments of social scientific theory. In turn, the theory will hopefully cast light on present diplomatic negotiation practices in ways that will promote peace more swiftly and more surely.

**062079.9**
TOWARD A SYNTHESIS OF SOCIAL PSYCHOLOGICAL "THEORIES", by, C. Slawski, *California State University, Long Beach, CA*, 29*pp.* ($2.50).

An attempt is first made to classify sixteen contemporary theoretical perspectives in social psychology by their ordinary disciplinary domain, namely, (A) sociology, (B) psychology, and (C) social psychology proper. A second classification concerns the typical research application, approach to studying human behavior, or the type of task engaged in by the social scientist, namely, (A) theorizing, (B) empirical research, or (C) participant observation. Then the most basic concepts and hypotheses of each perspective are related to a simple flow diagram in the mode of general systems theory in order to demonstrate the possibility for a larger synthesis of the various separate perspectives. The great number of disparate ad hoc perspectives in social psychology are in need of such synthesizing to give the discipline more breadth and generalizability to real life situations. Finally, a brief characterization of the perspectives (in five groupings) is given in order to present an overview and draw out the implied model of man-in-relationship.

**062279**
INVENTORY OF SYSTEMS RESEARCH IN THE NETHERLANDS (third, revised edition, April 1979), by Johannes van der Zouwen, *Vrije University, Amsterdam, The Netherlands*, 18pp. ($1.50).

This is an updated version of the description of systems research in the Netherlands which was listed under No. 060878.1 in the Depository (IJGS, Vol. 5, No. 2, p. 135).

**062679.01**
SYSTEMS RESEARCH MOVEMENT: CHARACTERISTICS, ACCOMPLISHMENTS AND CURRENT DEVELOPMENTS, a Report edited by Roger E. Cavallo and Sponsored by the Society for General Systems Research, July 1979, 132 pp. ($3.00).

This extensive report is the first "product" of an ongoing effort, sponsored by the Society for General Systems Research, to examine and characterize fundamental aspects of the contemporary systems movement and to make them accessible to as many people as possible.

Included are chapters on historical considerations, mainstream characterization of systems research, and suggestions for further work on the report.

**062679.02**
SUPPLEMENTARY MATERIAL TO THE SGSR REPORT ON SYSTEMS RESEARCH MOVEMENT (see GS Depositary entry No 062679.02), ($5.00).

As input to the task force report (see the previous entry in this Depository, No. 062679.01) some larger papers have been submitted which are being made available through this depository. This collection may be augmented by further material whose copyright status is not yet certain.

**092679**
PROCEDURES FOR GENERATING RECONSTRUCTION HYPOTHESE IN THE RECONSTRUCTABILITY ANALYSIS: APL VERSION, by Douglas Elias, *Dept. of Systems Science, School of Advanced Technology, State University of New York, Binghamton, New York, U.S.A.*

Based on the material presented in the paper "Procedures for generating reconstruction hypothese in the reconstructability analysis" by G. J. Klir and H. J. J. Uyttenhove (this issue, pp. xxx), this describes the construction and operation of APL-SV program for the refinement and aggregation for reconstruction hypotheses within the various classes. Described are the flow-of-control of the procedures within the reconstrauctability analysis package, listing and description of variables, and package-input-requirements.

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SEVENTH INTERNATIONAL CONGRESS OF MATHEMATICAL BIOLOGY
Paris, France. Theme of Congress: Mathematical Theory of Structure; Application to Sociological, Biological, Chemical and Physical Systems. *Contact* Societe Internationale de Biologie Mathematique, 11 bis Avenue de la Providence, 92160 Antony, France.

## September 10–13, 1979

INTERNATIONAL CONFERENCE ON SYSTEMS SCIENCE VI
Wroclaw, Poland. *Contact* Leszek Koszalka, Institute of Engineering Cybernetics, Technical University of Wroclaw, Janiszewskiego Street, 11/17, 50–370 Wroclaw, Poland.

## September 10–14, 1979

NINTH INTERNATIONAL CYBERNETICS CONGRESS
Namur, Belgium. *Contact* J. Lemaire, Managing Director, IAC, Palais des Expositions, Place Andre Rijckmans, B-5000, Namur, Belgium.

## October 3-5, 1979

FIFTH INTERNATIONAL CONFERENCE ON VERY LARGE DATA BASES
Rio de Janeiro, Brazil. *Contact* Y. W. Su, Computer and Information Sciences Dept., University of Florida, Gainsesville, Florida 32611, U.S.A.

## October 8–12, 1979

INTERNATIONAL SYMPOSIUM ON SYSTEM-MODELING-CONTROL
Zakopane, Poland. *Contact* Center of Electronic Computing, Technical University of Lodž, Ul, Piotrkowska 220, 90–369 Lodž, Poland.

## December 3–5, 1979

1979 WINTER SIMULATION CONFERENCE
San Diego, California. *Contact* Mitchell G. Spiegel, Deputy Director, Dept. of Air Force, FEDSIM/NA, Washington, D.C. 20330, U.S.A.

## January 3–4, 1980

THE 13th HAWAII INTERNATIONAL CONFERENCE ON SYSTEMS SCIENCES
Ilikai Hotel, Honolulu, Hawaii. *Contact* Ralph H. Sprague, Jr., College of Business Administration, University of Hawaii, 2404 Maile Way, Honolulu, Hawaii 96822, U.S.A.

## January 7–10, 1980

TWENTY FOURTH ANNUAL MEETING OF THE SOCIETY FOR GENERAL SYSTEMS RESEARCH
San Francisco, California. *Contact* Bela H. Banathy, Far West Laboratory for Educational Research and Development, 1855 Folson Street, San Francisco, California 94103, U.S.A. Symposia are scheduled on the following topics: General Systems Methodology, Systems Philosophy, Systems Technology, Homeostatic Systems, Optimal Systems, Oscillatory Systems, Adaptive Systems, Learning Systems, Developmental Systems, Linguistic Systems, Time and Dynamics, Universal Theories and Reductionism, and Psychotherapy.

## April 8–11, 1980

FIFTH EUROPEAN MEETING ON CYBERNETICS AND SYSTEMS RESEARCH
Vienna, Austria. *Contact* Robert Trappl, Österreichische Studiengesellshaft fur Kybernetik, Schottengasse 3, A-1010 Wien 1, Austria. The conference will include a Symposium on "General Systems Methodology". Papers submitted to this symposium should be sent to George J. Klir, Dept. of Systems Science, SAT, SUNY, Binghamton, New York 13901, U.S.A.

# DIGITAL PROCESSES

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AUTUMN-WINTER 1978

Editorial

Top-down Formal Specification and Verification of Parallel Control Systems
R. Valette and M. Diaz (Toulouse, France)

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An Algorithm for Designing Multiple Boolean Functions : Application to PLAs
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Sur la réalisation d'un multiplicateur ternaire 1 TRIT (On the Realization of a Ternary One TRIT Multiplier)
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R. Beaufils and J.P. Sansonnet (Toulouse, France)

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# NOTES FOR CONTRIBUTORS

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2. G. J. Klir, *An Approach to General Systems Theory* Van Nostrand Reinhold, New York, 1969.

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